# Unit-2 : Single Phase AC Parallel Circuit

## # A.C. Parallel Circuit :

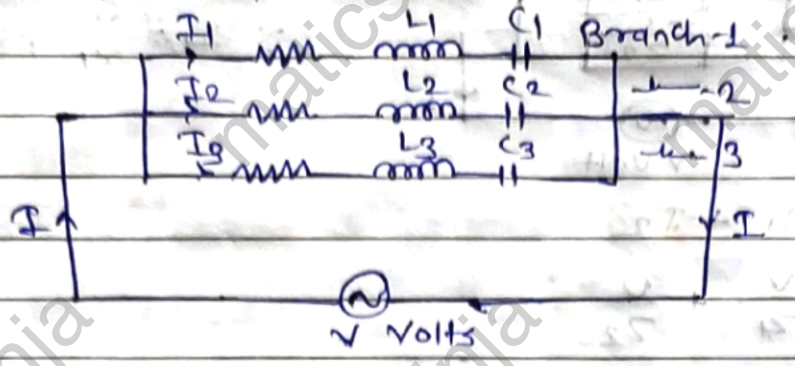



fig. Parallel AC circuit

Defn : A parallel ac circuit has two or more series circuits connected in parallel with each other across a common alternating voltage source as called as parallel AC ckt.

- Each series ckt is called as a branch in the parallel ckt.
- Let the current flowing through the three branches be $I_1$, $I_2$ & $I_3$ respectively. As the vtg drop across all the branches is the same i.e V Volts. These current are given by the following expression.

$$I_1 = \frac{V}{Z_1}, \quad I_2 = \frac{V}{Z_2} \quad \& \quad I_3 = \frac{V}{Z_3}$$

Applying KCL & using phasor Addition of we get,

$$\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$$

Substitute value of I we get,

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

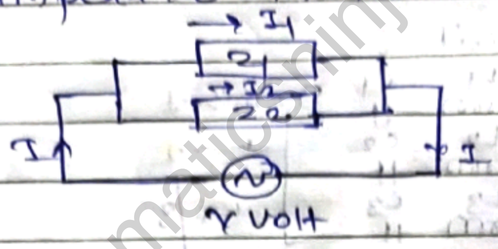
$$\frac{V}{Z} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$\frac{\cancel{V}}{Z \cdot \cancel{V}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\boxed{\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

Above eqn shows the equivalent ~~expression~~ Impedance of the parallel ckt.

# Two Impedance in Parallel :




$\therefore I_1 = \frac{V}{Z_1}$ & $I_2 = \frac{V}{Z_2}$

$$I = I_1 + I_2$$

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$\frac{V}{Z} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

$$\frac{\cancel{V}}{Z \cdot \cancel{V}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z} = \frac{Z_1 + Z_2}{Z_1 \cdot Z_2}$$

$$\boxed{Z_T = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}}$$

# Addition & Substraction of Impedance :

$$Z_1 = R_1 + jX_1$$
$$Z_2 = R_1 + jX_2$$

$$Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2)$$
$$= (R_1 + R_2) + j(X_1 + X_2)$$

Similarly

$$Z_1 - Z_2 = (R_1 + jX_1) - (R_2 + jX_2)$$
$$= (R_1 - R_2) - j(X_1 - X_2)$$

# Multiplication & Division of Impedance :

$$Z_1 \cdot Z_2 = \cancel{(R_1 + j)}\,(Z_1 \cdot Z_2) \angle \phi_1 + \phi_2$$

$$\frac{Z_1}{Z_2} = \frac{Z_1}{Z_2} \angle \phi_1 - \phi_2$$

Numerical on Addition, subtraction, Multiplication & Division

1) Find the value of $z = \frac{(3+j4) \cdot 5\angle 30}{(6+j8)}$

in rectangular form.

Soln :

$$z = \frac{(3+j4) \cdot 5\angle 30}{(6+j8)}$$

Convert $(3+j4)$ & $(6+j8)$ in polar form

$$\therefore \quad z = \frac{(5\angle 53.13) \times (5\angle 30)}{(10\angle 53.13)}$$

$$z = \frac{25\angle 83.13}{10\angle 53.13}$$

$$z = 2.5\angle 30 \qquad \therefore z = \boxed{2.16 + j1.25}$$

2) If $A = 4+j7$, $B = 8+j9$ & $C = 5-j6$ then calculate :

① $\frac{A+B}{C}$ ② $\frac{A \cdot B}{C}$ ③ $\frac{A+B}{B+C}$ ④ $\frac{B-C}{A}$

Soln :

① $\frac{A+B}{C} = \frac{(4+j7)+(8+j9)}{(5-j6)} = \frac{12+j16}{5-j6} = \frac{20\angle 53.13}{7.81\angle -50.19}$

$= 2.56\angle 103.32^\circ$

② $\frac{A \cdot B}{C} = \frac{(8\angle 60) \cdot (12\angle 48.36)}{7.81\angle -50.19}$ Converted into Polar

$= \frac{96\angle 108.36}{7.81\angle -50.19} = 12.29\angle 158.55^\circ$

③ $\frac{A+B}{B+C} = \frac{(4+j7)+(8+j9)}{(8+j9)+(5-j6)} = \frac{12+j16}{13+j3} = \frac{20\angle 53.13^\circ}{13.34\angle 12.99^\circ}$

$= 1.49\angle 40.14^\circ$

④ $\frac{B-C}{A} = \frac{(8+j9)-(5-j6)}{(4+j7)} = \frac{3+j15}{4+j7} = \frac{15.29\angle 78.69}{8.06\angle 60.25}$

$= 1.89 \angle 18.44^\circ$

3) find $z = \frac{(5+j8)}{(7+j9)}$ in Rectangular form.

soln: $z = \frac{(5+j8)}{(7+j9)} = \frac{9.43\angle 57.99}{11.40\angle 52.12}$

$= 0.82 \angle 5.87.$

$\boxed{z = 0.81 + j0.08}$

4) If $z_1 = 3+j7$ & $z_2 = 12-j16$ are connected in parallel find the equivalent Impedance of Combination.

Soln: Given, $z_1 = 3+j7$, $z_2 = 12-j16$

$z_1 = 3+j7 = 7.61 \angle 66.80$

$z_2 = 12-j16 = 20 \angle -53.13$

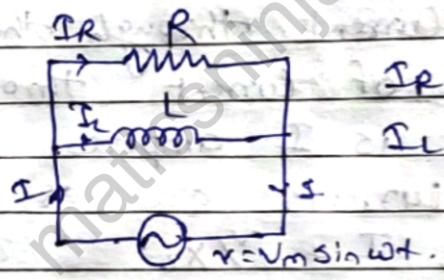
$z_{eq} = \frac{z_1 \cdot z_2}{z_1+z_2} = \frac{(7.61\angle 66.80)(20\angle -53.13)}{(3+j7)+(12-j16)}$

$= \frac{152.2\angle 13.67}{15-j9} = \frac{152.2\angle 13.67}{17.49\angle -30.96}$

$= 8.70 \angle 44.63$

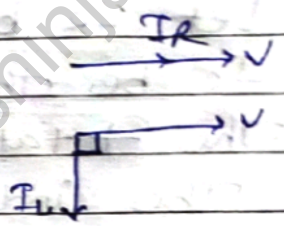
$\boxed{z_{eq} = 6.19 + j\,6.11}$

# R-L Parallel Circuit :




$I_R = V/R$

$I_L = V/X_L$

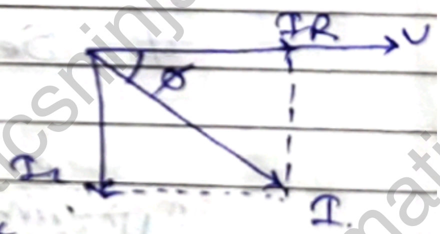



- In above fig. shows the parallel R-L circuit.
- The AC voltage source of instantaneous vtg $v = V_m \sin \omega t$ has been connected across the parallel combination of L & R.
- Let $I_R$ be the current through the resistance & $I_L$ be the current through the inductor. The total current is I amperes.
- The individual currents are as follows :

$$I_R = V/R \quad , \quad I_L = V/X_L$$

where, V = RMS value of supply vtg.

## * Phasor diagram :

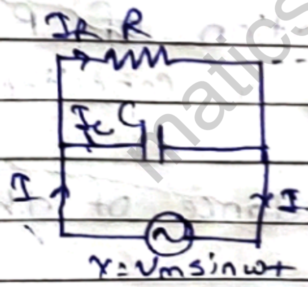



- The supply current I is equal to the phasor addition of $I_R$ & $I_L$

$$\therefore \bar{I} = \bar{I_R} + \bar{I_L}$$

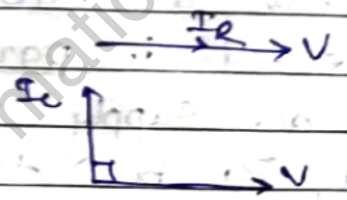
$$\therefore \frac{V}{R} + \frac{V}{X_L}$$

# R-C Parallel Circuit :




$I_R = V/R$

$I_C = V/X_C$



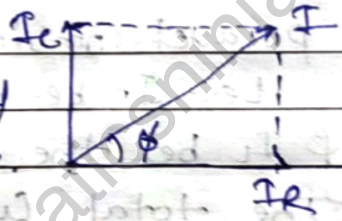

- In above fig. shows the parallel R-C circuit with an AC excitation.
- A pure capacitor of value C is connected in parallel with a resistor R & a voltage source of instantaneous vtg $v = V_m \sin \omega t$ is connected across the parallel combination of R & C.

- Let $I_R$ be the current through the resistance & $I_C$ be the value of current through capacitor. The total current is $I$ ampere.
- The individual currents are as follows:

$$I_R = V/R \, , \quad I_C = V/X_C$$

where, $V$ = rms value of supply vtg.

★ Phasor diagram:

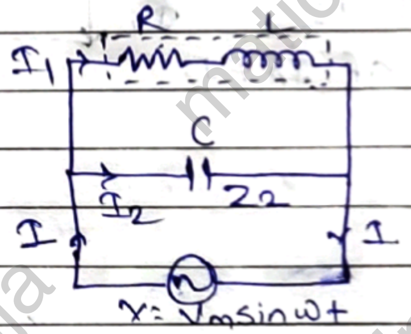



- The supply current $I$ is equal to the phasor addition of $I_R$ & $I_C$.

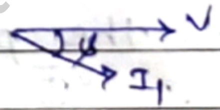
$$\bar{I} = \bar{I_R} + \bar{I_C}$$

$$= \frac{V}{R} + \frac{V}{X_C}$$

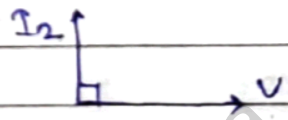
## # Series Combination of R & L in Parallel with Capacitor:




$$I_1 = \frac{V}{Z_1}$$




$$I_2 = \frac{V}{X_C}$$




- In above fig. shows the AC parallel circuit in which a series combination of R & L is connected in parallel with a capacitor & across the ac supply voltage.
- Let, $Z_1$ be the impedance of the RL series combination.

$$\therefore Z_1 = R + jX_L = |Z_1| \angle \phi_1$$

- Let $Z_2$ be the impedance of the branch containing the capacitor.

$$\therefore Z_2 = \cancel{0+j}\, 0 - jX_C = |Z_2| \angle \phi_2$$

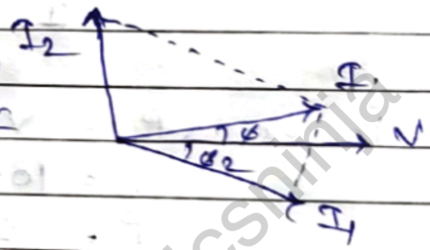
- The individual currents are given by

$$I_1 = \frac{V}{Z_1} \quad , \quad I_2 = \frac{V}{Z_2}$$

where, V = RMS Value of supply voltage.

★ **Phasor diagram :**

- The supply current I is equal to the phasor addition of $I_1$ & $I_2$.




$$\therefore \quad \bar{I} = \bar{I_1} + \bar{I_2}$$

$$= \frac{V}{Z_1} + \frac{V}{X_c}$$

Same as (4)

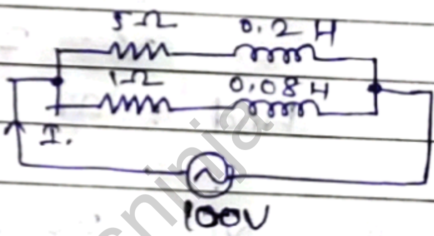
## Numerical :

1) A coil having resistance of 5 Ω & Inductance of 0.2 H is arranged in parallel with another coil having resistance of 1 Ω & Inductance of 0.08 H. Calculate the current through the combination & Power absorbed when a voltage of 100V, 50 Hz is applied. Use impedance method.

Sol$^n$ :

Coil 1 : $R_1 = 5\Omega$, $L_1 = 0.2\,H$

Coil 2 : $R_2 = 1\Omega$, $L_2 = 0.08\,H$




for coil 1 :

$$X_1 = 2\pi F L_1 = 2\times 3.14 \times 50 \times 0.2 = 62.84\,\Omega$$

$$Z_1 = R_1 + jX_1 = 5 + j62.84 = 63.03 \angle 85.45^\circ$$

for coil 2 :

$$X_2 = 2\pi F L_2 = 2 \times 3.14 \times 50 \times 0.08 = 25.12\,\Omega$$

$$Z_2 = R_2 + jX_2 = 1 + j25.12 = 25.13 \angle 87.72^\circ$$

$$Z_{eq} = \frac{Z_1 . Z_2}{Z_1 + Z_2} = \frac{(63.03 \angle 85.45) \times (25.13 \angle 87.72)}{(63.03 \angle 85.45) + (25.13 \angle 87.72)}$$

$$= \frac{1583.94 \angle 173.17}{(5+j62.84) + (1+j25.12)}$$

$$= \frac{1583.94\angle 173.17}{\cancel{88}.6 + j87.96} = \frac{1583.94\angle 173.17}{88.16\angle 86.09}$$

$$\boxed{Z_{eq} = 17.96\angle 87.09}$$

Total Current (I) $= \frac{V}{Z_{eq}} = \frac{100\angle 0^\circ}{17.96\angle 87.08}$

$\therefore I = 5.56\angle -87.09^\circ$ Amp.

Power (P) $= VI\cos\phi$

$= 100 \times 5.56 \times \cos(-87.08)$

$P = 28.3\ W$

$$\boxed{I = 5.56\angle -87.09^\circ \text{ Amp } \& \ P = 28.3 W}$$

Same as (5)

2) Two Impedance $Z_1 = 8 + j6\ \Omega$ & $Z_2 = 4 + j4$ are connected in parallel across 230V, 50 Hz AC supply. Calculate: (1) ~~...~~ Branch Current ($I_1$) (2) Branch Current ($I_2$) (3) Total Current ($I_T$)

* Draw the phasor diagram.

Soln:

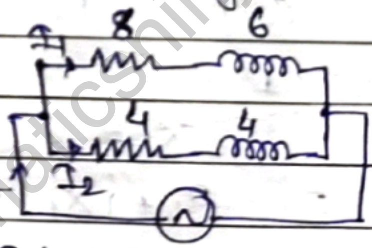
$Z_1 = 8 + j6 = 10\angle 36.85^\circ$

$Z_2 = 4 + j4 = 5.65\angle 45^\circ$

$V = 230V, \quad f = 50\ Hz$

Find: $I_1, I_2$ & $I_T$ with Phasor diagram

$I_T = \frac{V}{Z_{eq}} = \frac{230\angle 0}{3.61\angle 42.05} = 55.29\angle -42.05$




$Z_{eq} = Z_1 \| Z_2 = \frac{Z_1 . Z_2}{Z_1 + Z_2} = \frac{(10\angle 36.85) \times (5.65\angle 45)}{(8+j6) + (4+j4)}$

$= \frac{56.5\angle 81.86}{12 + j10}$

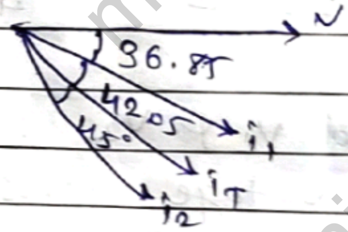
$= \frac{56.5\angle 81.86}{15.62\angle 39.8}$

$Z_{eq} = 3.61\angle 42.05^\circ$

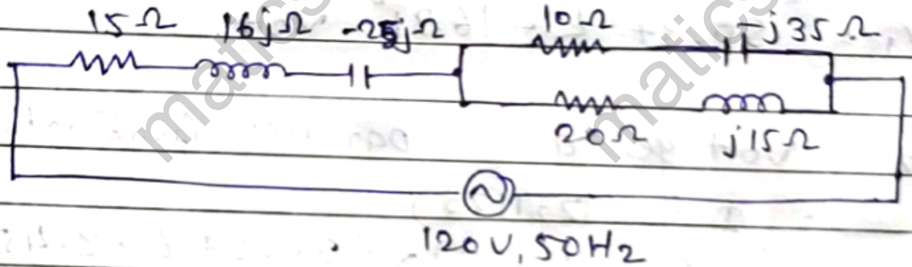
Branch Current $(I_1) = \frac{V}{Z_1} = \frac{200\angle 0}{10\angle 36.85} = 20\angle -36.85$ A

Branch Current $(I_2) = \frac{V}{Z_2} = \frac{200\angle 0}{5.65\angle 45} = 35.39\angle -45^\circ$ A.

Phasor diagram :




3) Find $I$, $V_1$ & $V_2$ & V for the circuit as shown in fig.




Soln : $V = 120V$,

Let $Z_1 = 15 + j(16-25)$

$= (15 - j9) = 17.49\angle -30.96^\circ\,\Omega$

$Z_2 = 10 - j35 = 36.4\angle -74^\circ\,\Omega$

$Z_3 = 20 + j15 = 25\angle 36.86\,\Omega$

Find Total impedance $Z$ :

$Z = Z_1 + (Z_2 \| Z_3)$ — ①

$$Z_2\|Z_3 = \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} = \frac{(36.49\angle -74)(25\angle 36.86)}{10 - j35 + 20 + j15}$$

$$= \frac{910\angle -37.14}{30 - j20}$$

$$= \frac{910\angle -37.14}{36\angle -33.69}$$

$\therefore Z_2\|Z_3 = 25.27\angle -3.45$

$Z_2\|Z_3 = 25.22 - j1.52$ — ②

Substituting Eqn ② in Eqn ① we get,

$Z = Z_1 + (Z_2\|Z_3)$

$= 15 - j9 + 25.22 - j1.52$

$\therefore \; Z = 40.22 - j10.52 = 41.57 \angle -14.65^\circ$

Find total Current (I) :

$I = \frac{V}{Z} = \frac{120 \angle 0^\circ}{41.57 \angle -14.65} = 2.88 \angle 14.65^\circ A.$

Find $V_1$ & $V_2$ :

$V_1$ = Voltage across $Z_1$

$= I \times Z_1$

$= (2.88 \angle 14.65) \times (17.49 \angle -30.96^\circ)$

$V_1 = 50.37 \angle -16.31^\circ$ Volt.

$V_2$ = Voltage across parallel Combination of $Z_2$ & $Z_3$

$= I \times (Z_2 \| Z_3)$

$= (2.88 \angle 14.65) \times (25.27 \angle -3.45)$

$V_2 = 72.77 \angle 11.2^\circ$ Volt.

4) A choke coil has a resistance of $2\Omega$ & an inductance of 0.035 H is connected in parallel with a 350 μF capacitor which is in series with a resistance of $20\Omega$. When the combination is connected across a 200V, 50 Hz supply.

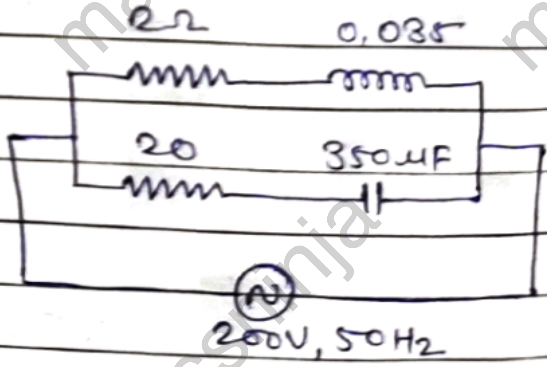
Calculate : ① Total Current taken

② Power factor of whole ckt.

Solⁿ : Given,

$R_1 = 2\Omega \quad L = 0.035$

$R_2 = 20\Omega \quad C = 350 \times 10^{-6} F$




$I = \frac{V}{Z_{eq}}$

$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$

$Z_1 = R_1 + jX_L$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.035 = 11\Omega$

$Z_1 = 2 + j11 = 11.18 \angle 79.69^\circ$

$Z_2 = R_2 - jX_C$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 350 \times 10^{-6}} \approx 9.09\Omega$

$Z_2 = 20 - j9.09 = 21.96 \angle -24.44$

$Z_{eq} = \dfrac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \dfrac{(11.18 \angle 79.69)(21.96 \angle -24.44)}{2 + j11 + 20 - j9.09}$

$= \dfrac{245.51 \angle 55.25}{22 + j1.91}$

$= \dfrac{245.51 \angle 55.25}{22.08 \angle 4.96}$

$\therefore Z_{eq} = 11.11 \angle 50.29^\circ$

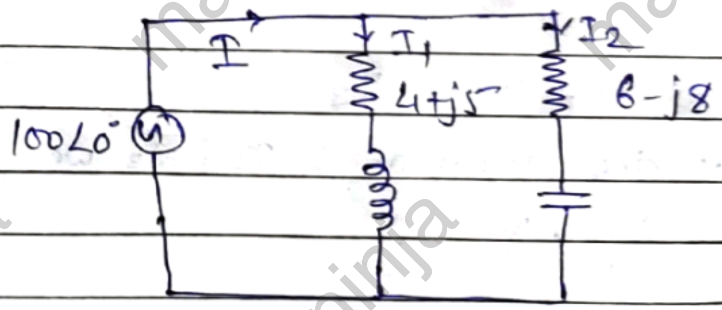
$I = \dfrac{200 \angle 0^\circ}{11.11 \angle 50.29} = 18 \angle -50.29 \text{ A}.$

Power factor $= \cos\phi$

$= \cos(-50.29)$

$= 0.63$ lagging

5) Calculate current $I_1$, $I_2$ & total current $I$ in the ckt shown.




Solⁿ :-

$Z_1 = 4 + j5 = 6.4 \angle 51.34^\circ$

$Z_2 = 6 - j8 = 10 \angle -53.13^\circ$

$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{(6.4\angle 51.34)(10\angle -53.13)}{4+j5+6-j8}$$

$$= \frac{64\angle -1.79}{10-j3} = \frac{64\angle -1.79}{10.44\angle -16.69}$$

$$\therefore Z_{eq} = 6.13\angle 14.9^\circ \ \Omega$$

$$I = \frac{V}{Z_{eq}} = \frac{100\angle 0}{6.13\angle 14.9} = 16.31\angle -14.9^\circ \ A$$

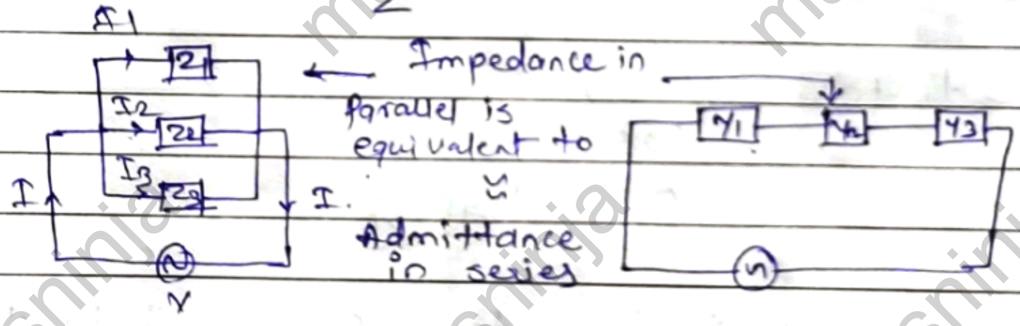
$$I_1 = \frac{V}{Z_1} = \frac{100\angle 0}{6.4\angle 51.34} = 15.62\angle -51.34^\circ \ A$$

$$I_2 = \frac{V}{Z_2} = \frac{100\angle 0}{10\angle -53.13} = 10\angle 53.13 \ A$$

## * The Concept of Admittance :

Def$^n$ : The admittance is defined as the reciprocal of the impedance.
It is denoted by Y & unit is Siemens (S).

$$\therefore Y = \frac{1}{Z}$$




(a) Impedance in Parallel (b) Admittance in series

$$I = I_1 + I_2 + I_3$$

Substitute the value of $I_1, I_2$ & $I_3$ we get,

$$I = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

Using Def$^n$ of Admittance

$$\therefore I = VY_1 + VY_2 + VY_3$$

$$I = V(Y_1 + Y_2 + Y_3)$$

Hence the effective admittance $\bar{Y}$ is given by,

$$\therefore \bar{Y} = \bar{Y_1} + \bar{Y_2} + \bar{Y_3}$$

* Conductance (G) : The conductance (G) is defined as the ratio of the resistance (R) & the square impedance ($Z^2$). It is denoted by (G) & It is measured in siemens

$$G = \frac{R}{Z^2}$$

* Susceptance (B) : Susceptance (B) is defined as the ratio of the reactance ± X & the squared impedance ($Z^2$).

It is denoted by (B) & measured in siemens. The susceptance can be inductive or capacitive depending on the polarity of X.

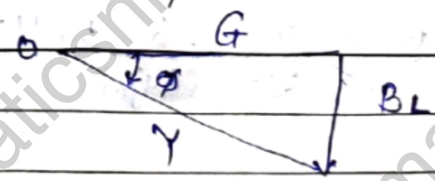
$$\therefore B = \frac{X}{Z^2}$$

- If the polarity of susceptance is positive it is said to be capacitive susceptance ($B_C$) & If the polarity is negative then the susceptance is said to be inductive susceptance ($B_L$).

| Polarity | + | − |
|---|---|---|
| Reactance (X) | Inductive | Capacitive |
| Susceptance (B) | Capacitive | Inductive |

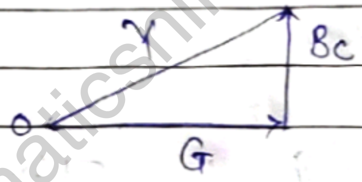
* Admittance Triangle :

- For Inductive Susceptance :




$$Y = G - jB$$

- For Capacitive Susceptance :




$$Y = G + jB$$

$Y^2 = G^2 + B^2$ $\quad \tan^{-1}\left(\frac{B}{G}\right)$

$Y = \sqrt{G^2+B^2}$

## * Solved Numerical by Admittance Method :

1) Calculate Conductance & Susceptance if $Z = 6+j8\,\Omega$

Soln :

$$Z = 6+j8 = 10\angle 53.13^\circ$$

$$Y = \frac{1}{Z} = \frac{1}{10\angle 53.13} = 0.1\angle -53.13^\circ$$

In Rectangular

$Y = 0.06 - j0.079$

$Y = G - jB$

Compair with above Eqn we get,

$G = 0.06$ & $B = -0.079$

or Conductance $(G) = \frac{R}{Z^2} = \frac{6}{(6+j8)^2} = \frac{0.06}{-106.}$

Suc. $(B) = \frac{X}{Z^2} = \frac{8}{(6+j8)^2} = 0.08\angle -106.26$

2) What is the Value of Admittance, Conductance & Susceptance if $Z = 4+j6$.

Soln :

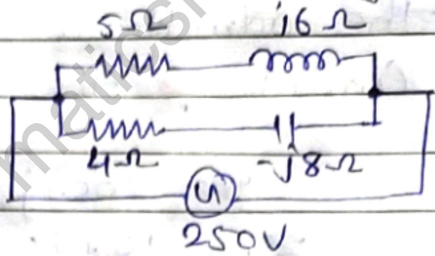
$$Z = 4+j6$$

$$Y = \frac{1}{Z} = \frac{1}{(4+j6)} = \frac{1}{7.21\angle 56.31} = 0.138\angle -56.31$$

$$\text{Conductance } (G) = \frac{R}{Z^2} = \frac{4}{(4+j6)^2} = 0.076\angle -112.61$$

$$\text{Susceptance } (B) = \frac{X}{Z^2} = \frac{6}{(4+j6)^2} = 0.115\angle -112.61$$

3) Find Admittance of each branch, Total admittance & total Current of the circuit shown in fig.




Sol$^n$:

Find : $Y, Y_1, Y_2$ & $I$

$Z_1 = 5+j6 = 7.81 \angle 50.19$

$Z_2 = 4-j8 = 8.94 \angle -63.43$

$\therefore Y_1 = \frac{1}{Z_1} = \frac{1}{7.81 \angle 50.19} = 0.128 \angle -50.19^\circ S$

$= 0.081 - j0.098$

$\therefore Y_2 = \frac{1}{Z_2} = \frac{1}{8.94 \angle -63.43} = 0.11 \angle 63.43^\circ S$

$= 0.049 + j0.098$

$Y = Y_1 + Y_2$

$= (0.081 - j0.098) + (0.049 + j0.098)$

$= 0.13 + j0$

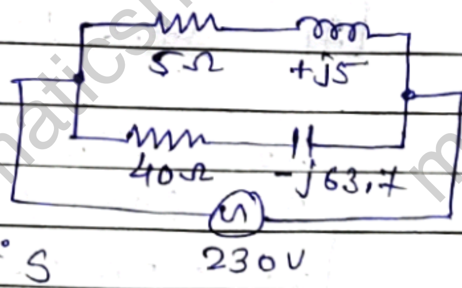
$\boxed{Y = 0.13 \angle 0^\circ S}$

$I = V.Y$

$= (250 \angle 0).(0.13 \angle 0^\circ)$

$\boxed{I = 32.5 \angle 0^\circ A.}$

4) Find the admittance of each branch ~~find total admitta~~ shown by circuit.




Sol$^n$:

$Z_1 = 5+j5 = 7.07 \angle 45^\circ$

$Z_2 = 40 - j63.7 = 75.21 \angle -57.87$

$Y_1 = \frac{1}{Z_1} = \frac{1}{7.07 \angle 45} = 0.141 \angle -45^\circ S$

$= 0.099 - j0.099$

$Y_2 = \frac{1}{Z_2} = \frac{1}{40-j63.7} = \frac{1}{75.21 \angle -57.87} = 0.013 \angle 57.87^\circ S$

$= 0.006 + j0.011$

$Y = Y_1 + Y_2 = (0.099 - j0.099) + (0.006 + j0.011)$

$Y = 0.105 - j0.088 S$ ← Ans in Rec. form

$Y = 0.137 \angle -39.96^\circ S$ ← Ans in Polar form

5) Impedance $Z_1 = (10+j5)$ ohm & $Z_2 = (8+j6)\Omega$ are connected in parallel across $V = (200+j0)$. Using the admittance method, calculate the circuit current & the branch currents.

Soln :

$$V = 200+j0 = 200\angle 0^\circ$$
$$Z_1 = 10+j5 = 11.18\angle 26.56^\circ$$
$$Z_2 = 8+j6 = 10\angle 36.86^\circ$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{11.18\angle 26.56} = 0.089\angle -26.56 = 0.08 - j0.039$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{10\angle 36.86} = 0.1\angle -36.86 = 0.08 - j0.059$$

$$Y = Y_1 + Y_2$$
$$= (0.08 - j0.039) + (0.08 - j0.059)$$
$$Y = 0.16 - j0.098$$
$$Y = 0.18\angle -31.48$$

$$\therefore I = V.Y$$
$$= (200\angle 0^\circ)(0.18\angle -31.48)$$
$$\boxed{I = 36\angle -31.48 \text{ A.}}$$

$$\therefore I_1 = V.Y_1$$
$$I_1 = (200\angle 0^\circ)(0.089\angle -26.56)$$
$$\boxed{I_1 = 17.8\angle -26.56^\circ \text{ A.}}$$

$$I_2 = V.Y_2$$
$$I_2 = (200\angle 0^\circ)(0.1\angle -36.86)$$
$$\boxed{I_2 = 20\angle -36.86^\circ \text{ A.}}$$

6) Two parallel impedances $Z_1 = (10+j8)\Omega$ & $Z_2 = (15-j10)\Omega$ are connected to 230V, 50 Hz AC Supply. Using admittance method, calculate branch current, total current & P.F of whole circuit.

Soln. :

Given, $Z_1 = (10+j8)\Omega$, $Z_2 = (15-j10)\Omega$, $V = 230V$, $f = 50Hz$

Find : $I_1$, $I_2$, $I$ & P.F

Find $I_1$ :

$$Z_1 = 10+j8 = 12.8\angle 38.65^\circ\ \Omega$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{12.8\angle 38.65} = 0.078\angle -38.65\ S$$

$$I_1 = V.Y_1 = (230\angle 0^\circ)(0.078\angle -38.65)$$

$$= 17.94\angle -38.65\ A$$

$$I_1 = 14 - j11.20\ A.$$

Find $I_2$ :

$$Z_2 = 15-j10 = 18\angle -33.69^\circ$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{18\angle -33.69} = 0.055\angle 33.69^\circ\ S$$

$$I_2 = V.Y_2 = (230\angle 0^\circ)(0.055\angle 33.69)$$

$$= 12.65\angle 33.69^\circ\ A.$$

$$I_2 = 10.52 + j7.01\ A.$$

Total Current : $I = I_1 + I_2$

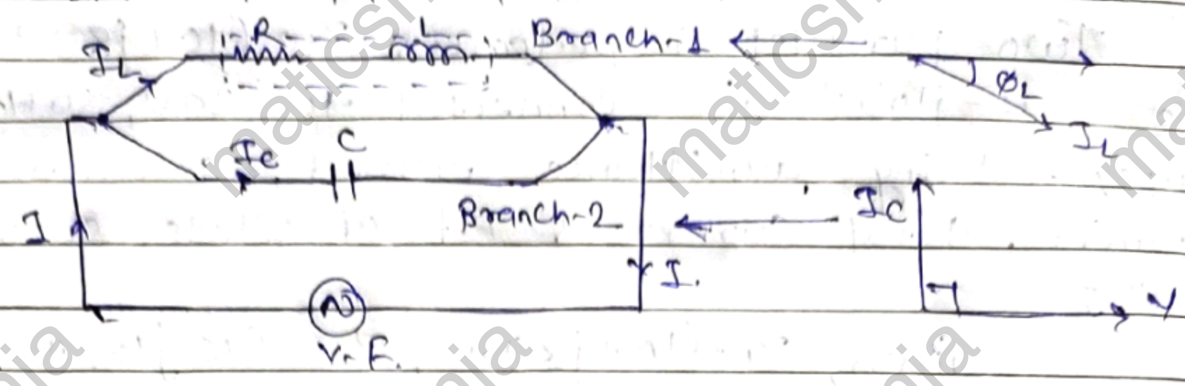
$$= (14-j11.20) + (10.52+j7.01)$$

$$I = 24.52 - j4.19\ A.$$

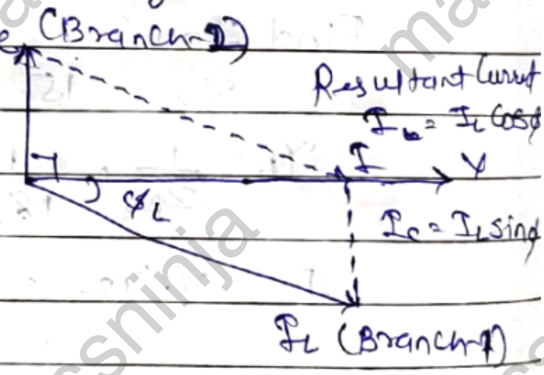
$$= 24.87\angle -9.69\ A$$

$$P.F = \cos\phi = \cos(\cancel{-4.19}\ -9.69) = 0.98\ (\text{lagging})$$

# Resonance in Parallel RLC Circuit :




- When applied voltage (V) & total current (I) are in phase then it is called as resonance frequency of parallel ac ckt.
- In above fig. shows the two branches are connected in parallel with each other.
- First branch consist of a resistance in series with a pure inductance, whereas the other branch consists of a pure capacitor.
- Let the sinusoidal voltage V of frequency f is connected across the parallel circuit. Let the current through the first branch is $I_L$ & that through second branch is $I_C$.




(fig. Complete Phasor Diag. at Resonance.)

- Combining the phasor dig. of individual branches we can draw the complete phasor dig. as shown in fig.

- At resonance V & I are in phase hence $\phi = 0$, Hence the power factor i.e $\cos\phi = 1$. Therefore the resonance condition is also called as unity P.f. Condition.

$$\therefore I = I_L \cos\phi_L$$

$$I_C = I_L \sin\phi_L$$

## * Expression for Resonance Frequency ($F_r$):

- At resonance, the value of current through branch-2 is given by,

$$I_C = I_L \sin\phi_L$$

But $I_C = \frac{V}{X_C}$, $I_L = \frac{V}{Z_L}$ & $\sin\phi_L = \frac{X_L}{Z_L}$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

$$\frac{\cancel{V}}{X_C} = \frac{\cancel{V} \cdot X_L}{Z_L^2}$$

$$\frac{1}{X_C} = \frac{X_L}{Z_L^2}$$

$$\therefore Z_L^2 = X_L \cdot X_C$$

$$\therefore R^2 + X_L^2 = \cancel{\omega} L \cdot \frac{1}{\cancel{\omega} C} = \frac{L}{C}$$ But $Z_L^2 = R^2 + X_L^2$

$$X_L^2 = \frac{1}{LC} - R^2$$

But $X_L = 2\pi F L$ & $2\pi F = \omega$

$\therefore X_L = \omega . L$

$$\omega L^2 = \frac{1}{LC} - R^2$$

$$\omega^2 . L^2 = \frac{1}{LC} - R^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$(2\pi F_r)^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$2\pi F_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{F_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

- If R is negligible we can neglect the second term in the above expression.

$$\therefore \boxed{F_r = \frac{1}{2\pi\sqrt{LC}}}$$

* **Dynamic Impedance at Resonance ($Z_D$) :**

The dynamic Impedance ($Z_D$) is defined as the impedance offered by the parallel R-L-C circuit at resonance.

• Expression for $Z_D$ :

- The current I at resonance is given by,

$$I = I_L \cos\phi_L$$

But, $I_L = \frac{V}{Z_L}$ & $\cos\phi_L = \frac{R}{Z_L}$

$$\therefore I = \frac{V}{Z_L} \times \frac{R}{Z_L}$$

$$I = \frac{V \cdot R}{Z_L^2}$$

But $Z_L^2 = R^2 + \omega^2 \cdot L^2$

$$I = \frac{V \cdot R}{R^2 + \omega^2 \cdot L^2}$$

~~For~~ But $R^2 + \omega^2 \cdot L^2 = L/C$

$$\therefore I = \frac{V \cdot R}{L/C}$$

$$I = \frac{V}{L/RC}$$

- The term $(L/RC)$ is called as the dynamic Impedance,

$$\therefore \boxed{I = \frac{V}{Z_D} \quad \& \quad Z_D = \frac{L}{RC}}$$ or $R_D$

## Q-Factor for Parallel R-L-C circuit :

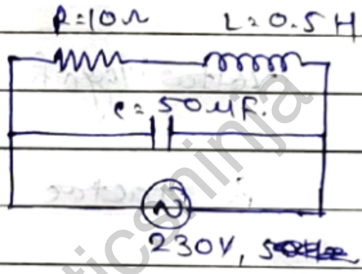
⇒ It is defined as the current magnification for provided at resonance called as Q-factor.

$$\boxed{\therefore \; Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

## Numerical on Resonance $f_r$, Q-Factor & $Z_D$.

1) An Inductive coil having resistance of $10\Omega$ & inductance of 0.5 H is connected in parallel with a capacitor of $50\mu F$, with 230 V. Determine :
   1) Parallel Resonance Frequency 2) Q-factor of Parallel ckt.

Soln :




Given, $R = 10\Omega$, $L = 0.5H$,
$C = 50\mu F = 50\times10^{-6} F$, $V = 230V$.
Find : $f_r$ & Q-Factor

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5\times50\times10^{-6}}} = 31.84\ Hz$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.5}{50\times10^{-6}}} = 10$$

2) A Coil with $R = 10\Omega$ & $L = 0.25$ H is connected in parallel with a capacitor of $100\mu F$. Find ;
   Q-Factor & Dynamic Resistance.

Soln :

Given, $R = 10\Omega$, $L = 0.25H$ & $C = 100\mu F = 100\times10^{-6}F$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.25}{100\times10^{-6}}} = 5$$

$$R_D = \frac{L}{RC} = \frac{0.25}{10\times100\times10^{-6}} = 250\ \Omega$$

* Comparison of Series & Parallel Resonant Ckt :

| Sr. No. | Parameter | Series Ckt | Parallel Ckt |
|---|---|---|---|
| 1) | Impedance | Min. $Z = R$ | Max. $Z_D = L/CR$ |
| 2) | Nature of Circuit | Resistive | Resistive |
| 3) | Power Factor | Unity (1) | Unity (1) |
| 4) | Current Drawn from the Source | Max. $I = \frac{V}{Z}$ | Min. $I = \frac{V}{(L/CR)}$ |
| 5) | Resonance frequency | $f_r = \frac{1}{2\pi\sqrt{LC}}$ | $f_r = \frac{1}{2\pi\sqrt{LC}}$ |
| 6) | ~~Voltage~~ Magnification | Voltage Magn. takes Place | Current magn. takes Place. |
| 7) | Q-Factor | $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ | $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ |
| 8) | Application | As an acceptor ckt | As a rejector ckt. |

Subject I/C
Mr. Bhosiwale G.H