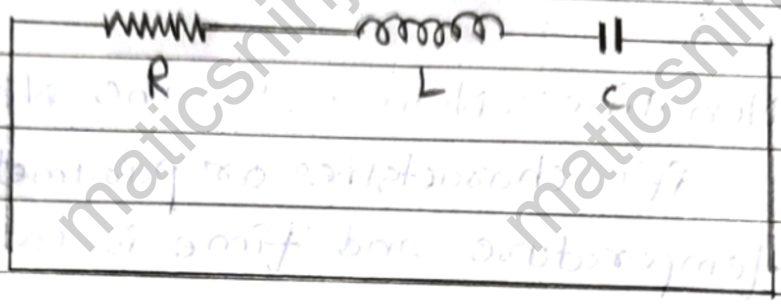
Unit : 1 Single Phase A.C. Series Circuits

Network :- All active elements or passive element when are electrically connected to each other it fomes a network.



Network

- Active Network → passive Network
- linear Network → Non linear Network
- Lamped Network → Distrubuted Network
- unilatral Network → Bilatral Network

1) Active Network :- The Network which contains any elergy source or active Element is called as Active element or Network

2) Passive Network :- The Network which does not contain any energy source or passive element is called as passive Network.

3) Linear NetWork :- The Network in which it's charactstics or parametars dose not change with time and temprature is called as linear Network.

4) Non-linear Network :- The Network in which it's charactstics or parameter changes it's temperature and time is called as Non linear Network.

5) Lumped Network :- The Network in which we can seprat it's element is called as Lamp Network.

6) Distrubuted Network :- The Network in which we can does not seprated it's element is called as Distrubuted Network

7) unilatral Network :- The Network in which it's charactstics does not change with change in direction of current is called as unilatral Network.

8) Bilatral Network :- The Network in which it's charactrstics change with change in direction of current is called as Bilatral Network.

★ Circuit Element :-

1) Resistor :-

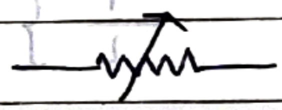
It is a material which oppose the flow of current through it.

Resistance :- It is the property of material which opposes the flow of current through it

$$R = \rho \frac{l}{a}$$

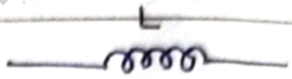
It's unit is Ω

R =  fixed Resistore

R =  Variable Resistor

★ Inductor :-

Inductor is a matarial which oppses change in current through it

Symbol for inductor 

$L = \frac{N \times \phi}{I}$ value is given by L

It's unit is Henry
it also measured in
$mH = 1 \times 10^{-3}$
$\mu H = 1 \times 10^{-6}$

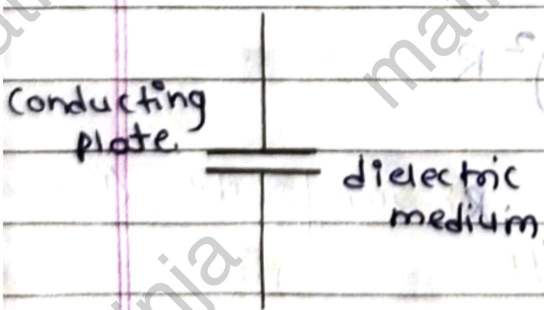
$V = L \frac{di}{dt}$

$i = \frac{1}{L} \int v dt$

current through a inductor

★ Capacitor :-

Capacitor is a electronic device which ~~focus~~ stored the electrical energy in the fome of static charge



It is denoted by C

and it's SI unit is Ferad.

Capacitons :-

Is a ability of capacitor to stored the a electrical charge

It's unit is farad.

It is given by

$$C = \frac{Q}{V}$$

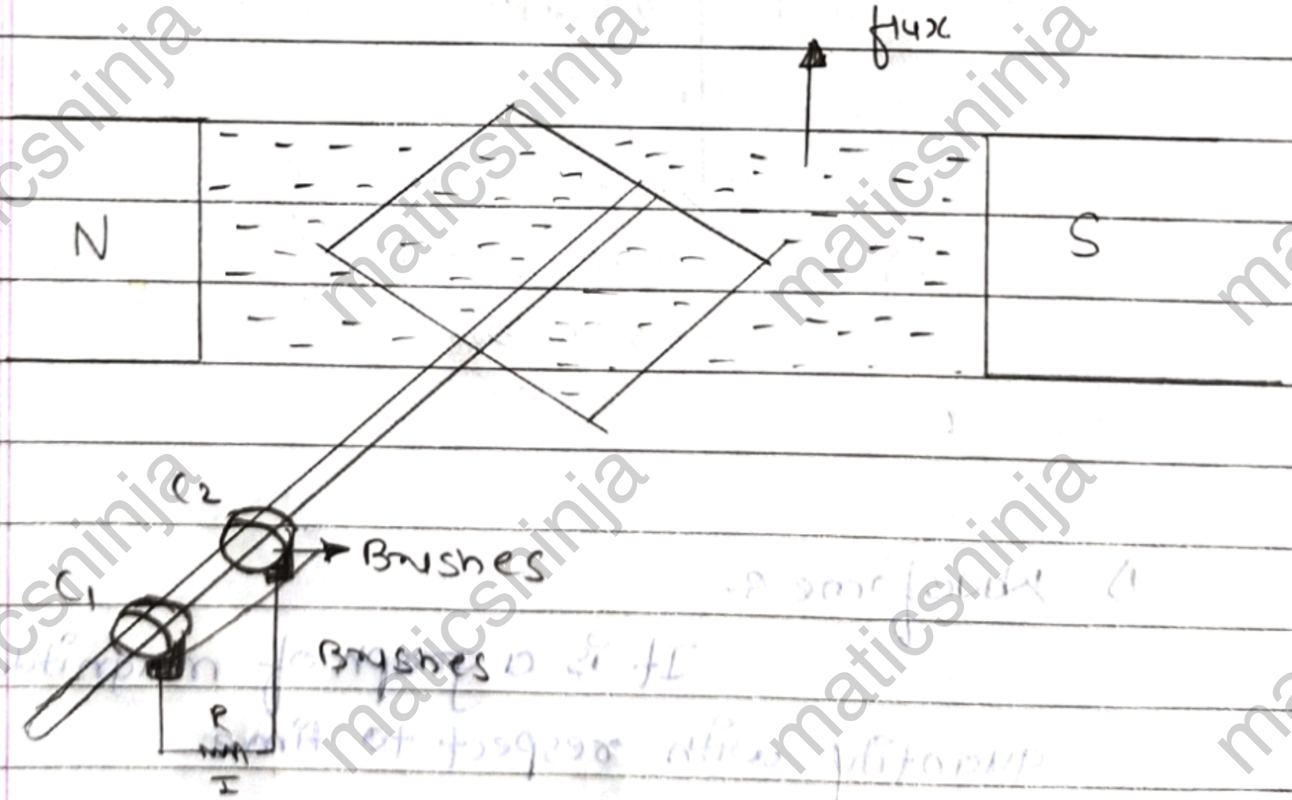
$$Q = CV$$

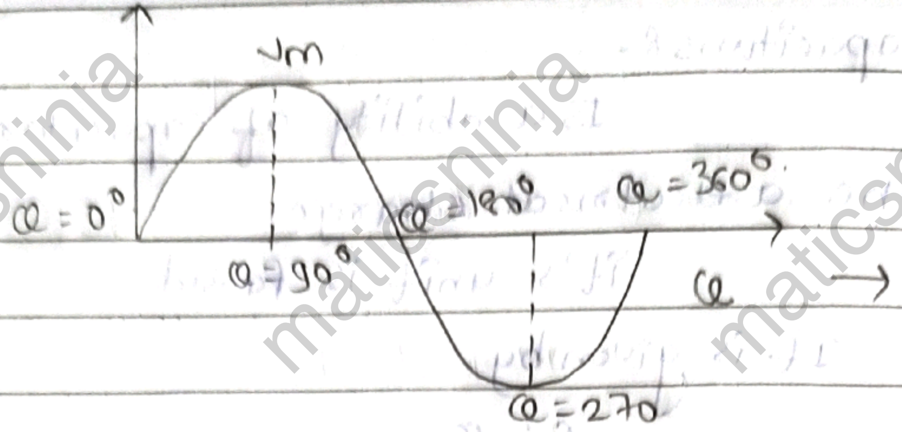
eqn for current $= i = \frac{dQ}{dt} = C\frac{dv}{dt}$

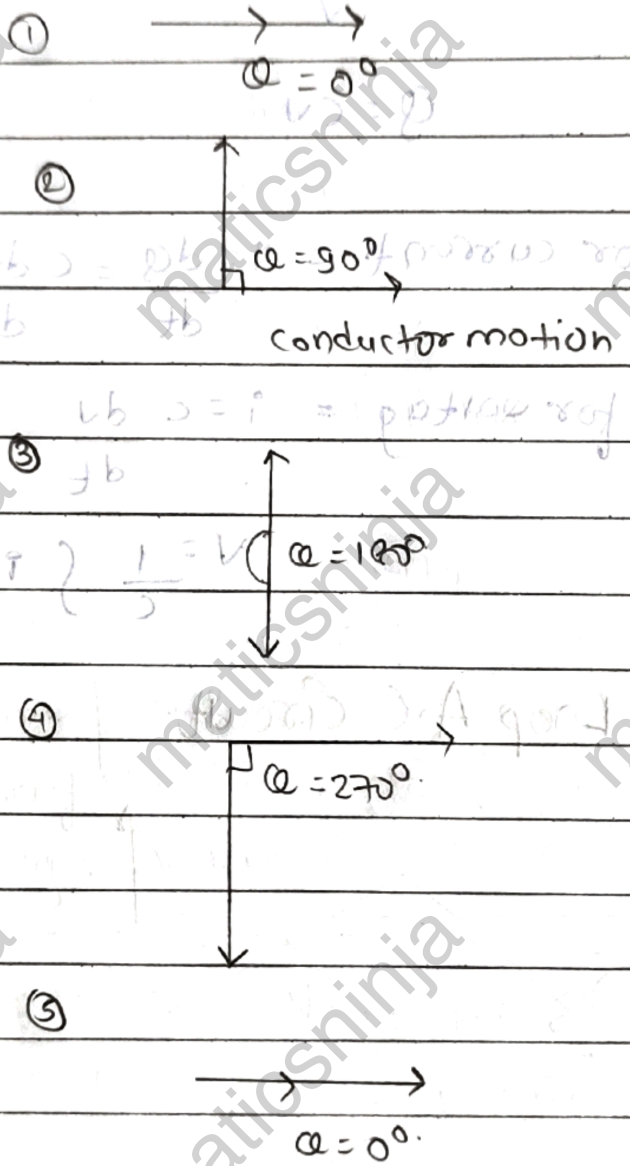
eqn for voltag $= i = C\frac{dv}{dt}$

$$V = \frac{1}{C}\int i\,dt$$

★ Simple Loop A.C Circuit.







1) Waveforme :

It is a graph of magnitude of AC quantity with respect to time

2) Instantaneous value :

It is the value of AC quanti

at perticular instant

3) Amplitude and peak value :-
It is a maximum value of peak value of AC quantity

4) Cycle :
Complet set of one positive halph cycle and one negative half cycle

5) frequency : The Number of cycles per second is frequency (Hz)

6) Time period :- Time require to complet one cycle is time period

7) RMS value :-
RMS valyue or effective value is equal to Stadystat current or DC current which produce same amount of heat as produced by AC current with temprature and resistance Remains constant

RMS value = $0.707 V_m$

or $\frac{V_m}{\sqrt{2}}$

od. M.
① ② $0.707 \times V_m$.
sq. Avg.
sq. root.
RMS.

8) Arrage value :-
It is a arrage of all intantonius value taken over a one halph cycle

V_{AVG} : $0.637 V_m$

9) Peak factor or Crest factor :-

It is the ratio of peak vale and Rms value is a peak factor

$$\frac{\text{Peak value}}{\text{Rms value}}$$

$$= \frac{V_m}{V_{Rms}}$$

$$= \frac{1 \cancel{V_m}}{0.707 \cancel{V_m}}$$

$$= 1.414.$$

10) ~~fome~~ form factor :-

It is a Ratio of Rms value to avrage value

$$\frac{V_{Rms} \text{ value}}{\text{avrage value}}$$

$$= \frac{V_{Rms}}{V_{Avr}}$$

$$= \frac{0.707 \cancel{V_m}}{0.637 \cancel{V_m}}$$

$$F.F. = 1.109.$$

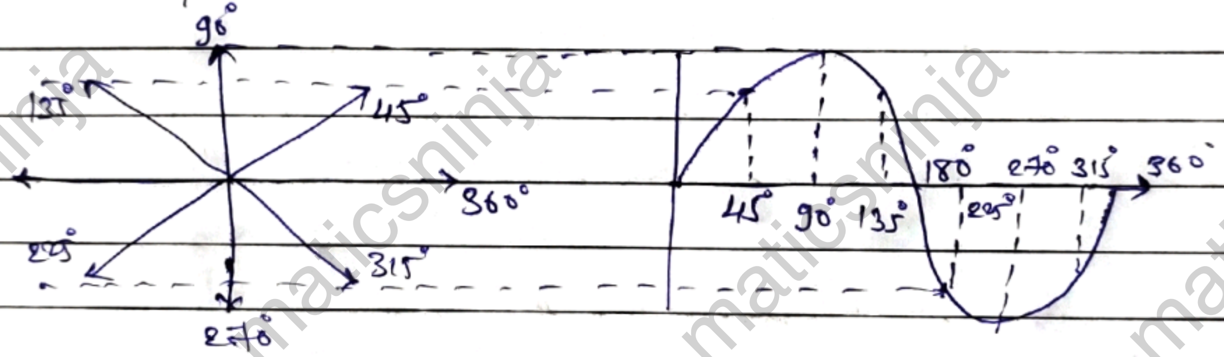
1) What is the Average value & RMS value of sinusoidal Alternating current of 31 A maximum value.

soln: Given, $I_m = 31\,A$.

$$I_{Avg} = 0.637 \times I_m$$
$$= 0.637 \times 31$$
$$I_{Avg} = 19.74\,A.$$

$$I_{rms} = 0.707 \times I_m$$
$$= 0.707 \times 31$$
$$= 21.91\,A.$$

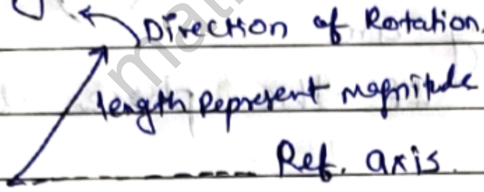
* Phasor Representation :



- It is a straight line with an arrow mark on a one side, the length of this straight line represent the magnitude of the sinusoidal quantity & the arrow represent its direction.

* Phasor Representation of an Alternating Quantity

① Phase Angle (θ) :



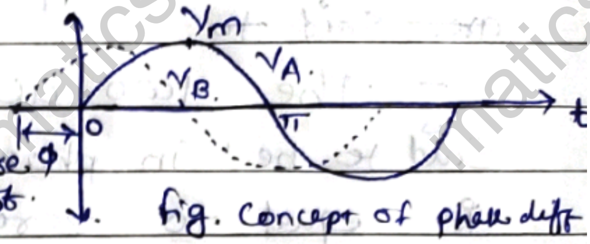


fig. Concept of phase diff

- The angle made by the conductor w.r.t. the magnitude Reference axis is called as phase Angle

② Phase Difference :

The phase diff. is the difference in the phase angle of two ~~AC~~ Sinusoidal AC waveform.

- The angle φ is known as the Phase difference betn V_A & V_B. It is measured in 'Radians'.

Types of Phase difference :

① Leading Phase diff. ② Lagging Phase diff.
③ In phase diff.

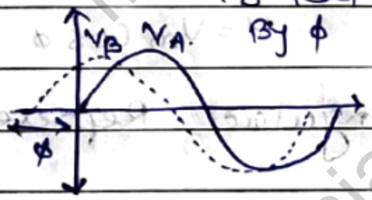
① Leading Phase diff. :-

If the phase angle φ is positive then the phase diff. φ is said to be a leading phase diff. In other words we say that Voltage V_B leads the Vtg. V_A.

V_B lead V_A By φ

$V_A = V_m \sin\theta$

$V_B = V_m \sin(\theta + \phi)$



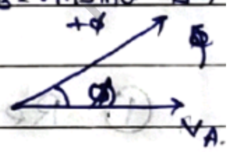


fig. Concept of leading Phase diff

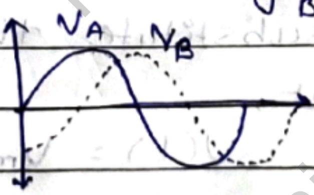
② Lagging Phase diff. :

If the phase angle φ is Negative then the phase diff. φ is said to be a lagging phase diff. In other words we say that Vtg V_B lags Vtg V_A.

V_B lags V_A By φ

$V_A = V_m \sin\theta$

$V_B = V_m \sin(\theta - \phi)$



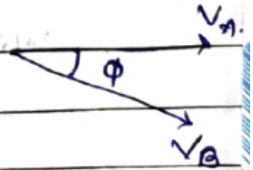
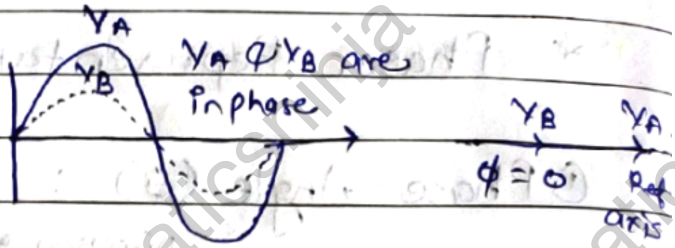


fig. Concept of lagging Phase diff

③ In phase diff. :

- ~~The two vtg V_A & V_B are said to.~~

- The two ac vtg are said to be "in phase" If the phase diff. bet^n them is equal to zero.

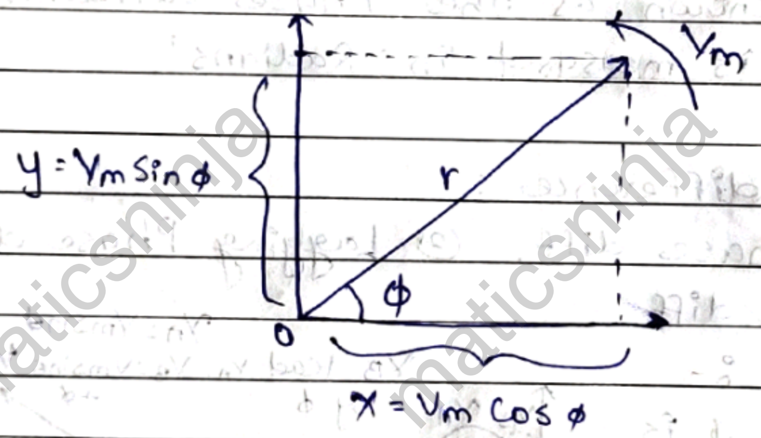


Single Phase ~~AC Circuit~~ :

Mathematical Representation of Phasor :-

- A phasor can be presented in two diff. ways:

① Rectangular form ② Polar form



① Rectangular Representation :

$$V(t) = x + jy \quad \text{--- ①}$$

x = x Component of the phasor = $V_m \cos\phi$

y = y component of the phasor = $V_m \sin\phi$

Substitute the value of x & y in eq^n ① we get,

$$V(t) = V_m \cos\phi + j V_m \sin\phi$$

e.g. If $V(t) = 20 \sin(100\pi t + 60^\circ)$ can be represent in the rectangular form as follow

$$V(t) = (20 \cos 60^\circ + j\, 20 \sin 60^\circ)$$
$$= 10 + j\, 17.32$$

② Polar Representation :

– the instantaneous vtg $v(t) = V_m \sin(\omega t + \phi)$ can be represented in the polar as follow :

$$v(t) = r \angle \phi$$

where $r = V_m$

e.g. $v(t) = 20 \sin(100\pi t + 60^\circ)$ is represented in the polar form as,

$$v(t) = 20 \angle 60^\circ \text{ volt.}$$

* Conversion from polar to Rectangular :

$$v(t) = r \angle \phi \qquad x = r\cos\phi,\ y = r\sin\phi$$

$$v(t) = r\cos\phi + r\sin\phi$$

e.g. $z = 10 \angle 60^\circ$

$$z = 10\cos 60^\circ + j\,10\sin 60^\circ$$

$$z = 5 + j\,8.66$$

* Conversion from Rectangular to Polar :

$$v = x + jy \qquad r = \sqrt{x^2+y^2},\ \theta = \tan^{-1}(y/x)$$

$$r = \sqrt{x^2+y^2} \quad \& \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

e.g. $16 + j8$

$$r = \sqrt{(16)^2 + (8)^2}$$

$$r = 17.88 \qquad \theta = 26.56$$

$$\boxed{17.88 \angle 26.56}$$

e.g. or Numerical :

1) Convert $z = 6 + j8\,\Omega$ in polar form

soln : $z = 6 + j8$ ..... in Rectangular form

$\therefore x = 6,\ y = 8$

$$r/z = \sqrt{x^2+y^2} = \sqrt{6^2+8^2} = 10$$

$\theta = \tan^{-1}(y/x) = \tan^{-1}(8/6) = 53.13°$

$\therefore\ z = 10\angle 53.13°$ .... Polar form

2) Convert $z = 16 + j8$ in Polar form $= \underline{17.89\angle 26.57°}$

3) convert $z = 20 - j34.64$ in Polar form $\simeq \underline{40\angle -60°}$

H.W $z = 4 + j5$ in Polar $= 6.40\angle 51.31$

$30\angle 60°$ in Rec. $= 15 + j25.98$

Rectangular

① Convert $z = 10\angle 60°$ in Rectangular form =

Solⁿ: $z = 10\angle 60°$

$= 10\cos 60 + j\,10\sin 60$

$z = 5 + j\,8.66\,\Omega$ .... Rectangular form

② Convert $z = 18\angle 30°\,\Omega$ in Rect. form $= \underline{15.58 + j9}$

③ Convert $z = 20\angle 60°$ in Rec. form $= \underline{10 + j17.32}$

Addition & Substraction by Using Rect. form

$V_1 = x_1 + jy_1$

$V_2 = x_2 + jy_2$

Add: $(V_1 + V_2) = (x_1 + x_2) + j(y_1 + y_2)$

Similarly

Sub: $(V_1 - V_2) = (x_1 - x_2) + j(y_1 - y_2)$

Multiplication & Division by using Polar form:

$V_1 = r_1\angle\phi_1$

$V_2 = r_2\angle\phi_2$

Multi. $(V_1 \cdot V_2) = (r_1 \cdot r_2)\angle\phi_1 + \phi_2$

Div $\dfrac{V_1}{V_2} = \dfrac{r_1}{r_2}\angle\phi_1 - \phi_2$

Note: For Addition & Subtraction, represent the phasors in their rectangular form.
For Multiplication and Division represent the phasors in their Polar form.

* Single Phase AC Circuits

- The three Basic elements of any AC circuit are Resistance (R), Inductance (L) & Capacitance (C).
- We are going to discuss the following three ac circuit first and then their Combination.
- The three basic AC circuit are:

① Purely Resistive AC Circuit
② Purely Inductive AC circuit
③ Purely Capacitive AC circuit

~~To Under these AC circuit we should have the to understood Reactance & Impedance.~~

⊛ Reactance and Impedance :

- In the circuit we have defined 'Resistance' as opposition to the flow of current. Similarly for a.c circuit we defined two terms namely '~~Resist~~ Reactance' (X) & Impedance (Z).

Reactance : Reactance can be of two types:
① Inductive Reactance (X_L)
② Capacitive Reactance (X_C)

① Inductive Reactance (X_L) :

Inductive Reactance defined as the opposition to the flow of an alternating current offered by an inductance.

It is denoted by X_L & Measured in (Ω)

$X_L = \omega L$

$\therefore \boxed{X_L = 2\pi f L}$

② Capacitive Reactance (X_C) :

The capacitive Reactance is defined as the opposition to flow of current offered by a pure capacitor. ~~to~~ +

It is denoted by X_C

The expression for capacitive Reactance is,

$X_C = \frac{1}{\omega C}$; $\boxed{X_C = \frac{1}{2\pi f C}}$

③ Impedance (Z) :

~~The~~ An Impedance is defined as the opposition offered by combined effect of resistance and reactance to the flow of alternating current.

It is denoted by Z

Impedance can be expressed in the Polar form as follows

$Z = |Z| \angle \phi \ \Omega$

where $|Z|$ = Magnitude of Z

ϕ = Phase Angle.

It is expressed in the Rectangular form as follow :

$Z = R + jX$

Where, R = Resistive Part

X = Reactive Part

Numerical :

1) Find out Resistance and capacitance of the given impedance. 1) $25\angle -45^\circ \Omega$

2) $10 - j15 \Omega$

Soln : 1) ~~10 - j15 Ω~~ $Z = 25\angle -45^\circ$

Convert polar to Rectangular

$R = Z\cos\phi = 25\cos(-45^\circ) = 17.67 \Omega$

$X_C = jZ\sin\phi = j\,25\sin(-45^\circ) = -17.67 \Omega$

$\therefore Z = R - jX_C$

$Z = 17.67 - j17.67$ ... Rect. form

$X_C = \dfrac{1}{2\pi fC}$ $\quad C = \dfrac{1}{2\pi \times 50 \times 17.67}$

$X_C = \dfrac{1}{2\pi f X_C}$ $\quad C = 0.000180 f$.

$C = 180.23 \mu f$

$\therefore R = 17.67 \Omega$ & $C = 180.23 \mu f$.

2) $10 - j15 \Omega$.

Soln : Compair $10 - j15$ with ~~zerrgiseo~~ $Z = R - jX_L$

we get, $R = 10 \Omega$.

$X_c = 15$

$X_c = \dfrac{1}{2\pi fC}$ $\quad C = \dfrac{1}{2 \times 3.14 \times 50 \times 15}$

$C = \dfrac{1}{2\pi fc}$ $\quad C = 0.00021231 f$.

$C = 212.31 \mu f$.

1. Purely Resistive AC ckt :

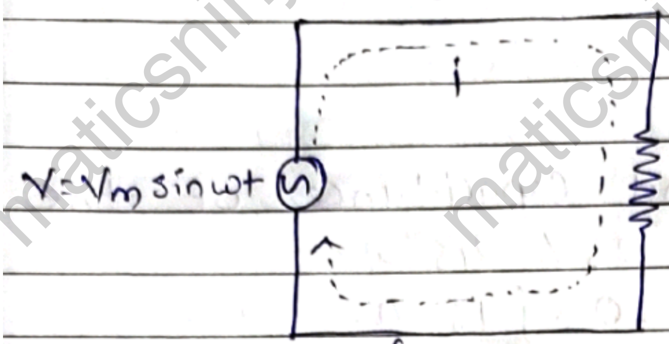


fig. (a)

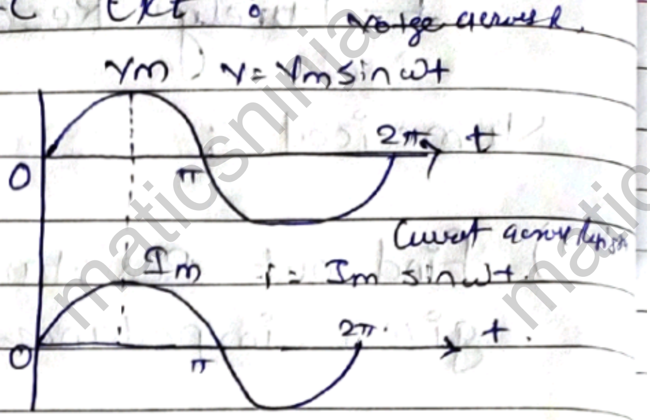
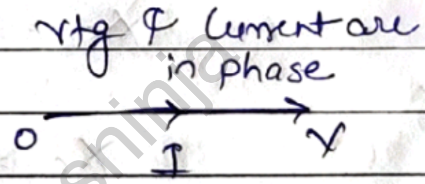


fig. (b)

- The pure resistive ac ckt is as shown in above fig.
- It consist of an ac voltage source $v = V_m \sin \omega t$ & a Resistor R connected across it.
- The instantaneous current flowing through the ckt is i.

* Phasor Diagram :



- The phase diff. betn the vtg & current phasor is $\phi = 0$
- So, the vtg & current are in phase.

* Vtg & Current Equation :

- Reffering to fig. (b) the instantaneous vtg across resister (v_R) is same as the source vtg.

$\therefore \; v_R = v = V_m \sin \omega t$.

- Applying the ohm's law the expression for instantaneous current flowing through resistor is given by,

$$i = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m \angle 0^\circ}{R \angle 0^\circ}$$

Let, $I_m = \frac{V_m}{R}$, $I = I_m \angle 0^\circ$

$\therefore \; i = I_m \sin(\omega t)$

* Impedance of the Purely Resistive ckt :

- Impedance in Rec. form

$$Z = R + jX$$

X → Reactive Part.

R → Resistive Part.

- When the load is Purely Resistive then the reactive Part is zero i.e $X = 0$, Hence the impedance of Resistive ckt is given by.

$$Z = R\,\Omega = (R + j0)\,\Omega$$

- In Polar form

$$Z = R\angle 0^\circ\,\Omega$$

* Power through Purely Res. ckt :

$$P = V_{rms}\,I_{rms}$$

Thus, the power in resistive ckt is equal to the product of rms value of vtg & current.

Q.1) Write the expression for instantanious current through pure ~~resistive~~ Resistance of value $100\,\Omega$ If the source vtg is $v(t) = 325\sin(100\pi t + \pi/3)$

Sol^n :-

$$v(t) = V_m \sin(\omega t + \phi)$$

$$= 325 \sin(100\pi t + \pi/3)$$

$$i = I_m \sin(\omega t + \phi)$$

$$I_m = \frac{V_m}{R} = \frac{325}{100} = 3.25\,A.$$

$$\therefore I(t) = I_m \sin(\omega t + \phi)$$

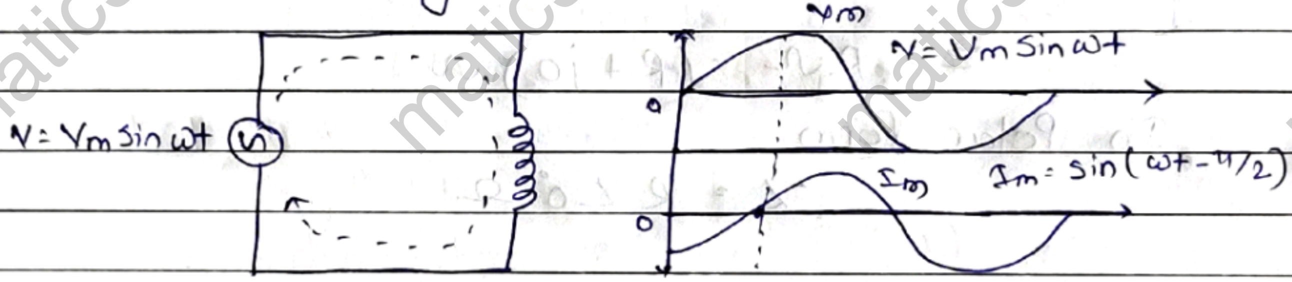
$$\boxed{I(t) = 3.25 \sin(100\pi t + \pi/3)}$$

Q.2) $v(t) = 100 \sin(314t + \pi/2)$, $R = 10\,\Omega$

Sol^n:
$v = V_m \sin(\omega t + \phi)$
$v = 100 \sin(314t + \pi/2)$
$i = I_m \sin(\omega t + \phi)$
$\therefore I_m = \frac{V_m}{R} = \frac{100}{10} = 10\,A$

$$\boxed{i(t) = 10 \sin(314t + \pi/2)}$$

2. Purely Inductive AC Circuit :



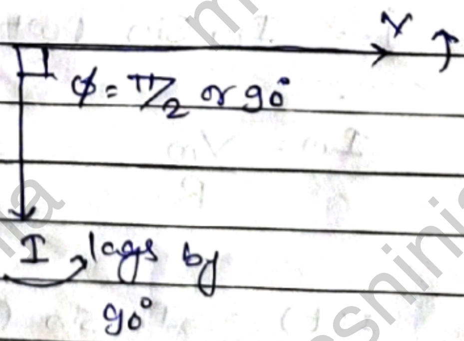
Current lag V by 90°

* V & Current eq^n :
$v = V_m \sin \omega t$, $\cancel{X_L} = \cancel{\frac{\phi}{I}}$, $\phi = \frac{\cancel{X_L}}{\cancel{2\pi f}}$
$I_m = \frac{V_m}{X_L}$, $X_L = 2\pi f L$

$i = I_m \sin\left(2\pi f t - \frac{\pi}{2}\right)$

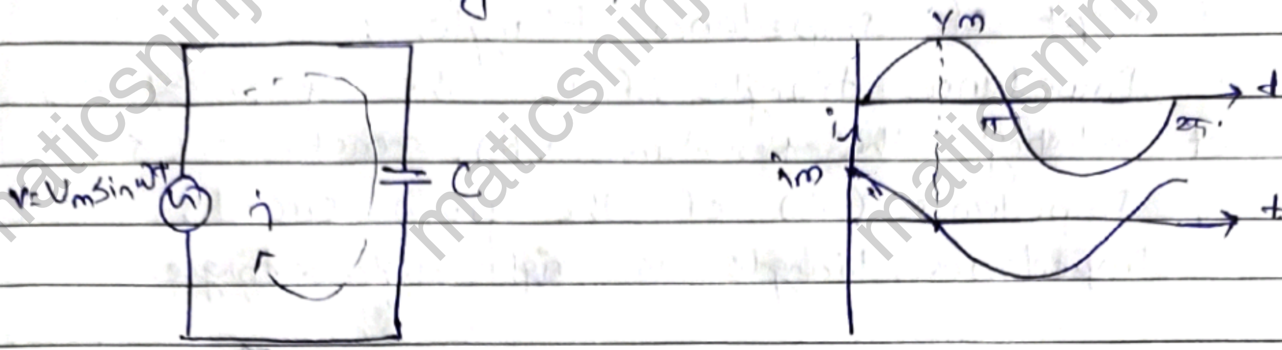
- i → instantaneous Current
- $\frac{\pi}{2}$ → Phase angle, $\pi/2$ radians or 90°
- $-$ → Current lag behind V
- f → frequency of current waveform
- $\sin$ → Current wave is sine wave
- I_m → Peak value

* Phasor diagram :



* Impedance of a Pure Inductive ckt :
- In Rectangular form : $Z = R + jX_L$: $Z = jX_L$
- In Polar form : $Z = X_L \angle 90°$

3. Purely Capacitive AC Circuit :



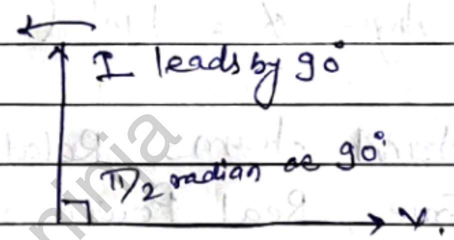
Vtg & current eqn.

$v = V_m \sin \omega t$

$I = I_m \sin(\omega t + \pi/2)$ → Current leads vtg

$I_{RMS} = \frac{V_{rms}}{X_C}$

Phasor diagram :



Impedance : $Z = R - jX_C$

Rec. form $R = 0$.

$\therefore Z = -jX_C$

In Polar form $Z = X_C \angle -90°\ \Omega$.

Ex. ① Find the current through a purely capacitive circuit containing a vtg source $v = 230V$, $50 Hz$ & capacitor of $1000 \mu F$.

Soln: RMS value of voltage $= 230V$, $f = 50$, $C = 1000 \mu F$.

find : RMS value of current.

RMS value of current $= \frac{V}{X_C}$

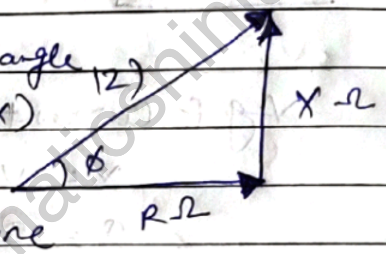
$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 1000 \times 10^{-6}} = 3.183\ \Omega$

$I = \frac{V}{X_C} = \frac{230}{3.183} = 72.26\ A.$

Unit-1

Chapter : 2 Single Phase AC Series Circuit

* Concept of Impedance Triangle :
  - Impedance triangle of is the graphical way of relating the Resistance (R), reactance (X), & Impedance (Z) of the given circuit.
  Impedance triangle is right angle triangle.



- The two side of Impedance triangle correspond to Resistance (R) & Inductance (X) with angle of 90°, betn them third side the correspond to magnitude of impedance |Z| with $|Z|^2 = R^2 + X^2$

$$R = |Z| \cos\phi, \quad X = |Z| \sin\phi$$

- Phase Angle $\phi = \tan^{-1}(X/R)$

* Important terms Related to Power
1. Active, True, Real Power (P)
2. Reactive, or imaginary Power (Q)
3. Apparent Power (S)
4. Power factor

1) Active Power : The truth power, real power & active power is defined as the Average power P_{av} consumed by the a.c circuit is called as active power.
It is denoted by P

$$P = P_{av} = VI\cos\phi \text{ watts.}$$

ϕ = Angle betn Vtg & Current

2) Reactive Power : Reactive Power is defined as the product of V & I with sine of angle betn V & I i.e ϕ.
It is Denoted by Q

$$Q = VI\sin\phi \quad \text{VAR}$$

It is measured in VAR - Volt-Amp. Reactive or KVAR

3) Apparent Power : Apparent power is defined as the product of the rms value of Vtg & Current. It is denoted by S

Measured in Volt-Amp. (VA) or kilovolt-Amp.

$\therefore \quad S = V \times I$

- It can be proved that apparent power is equal to the vector sum of active & reactive power.

$S^2 = P^2 + Q^2 \qquad S = \sqrt{P^2 + Q^2}$

$= \cancel{\sqrt{V^2 I^2 \cos^2\phi + V^2 I^2 \sin^2\phi}}$

4) Power Factor : It is the Ratio of true power & the apparent power. & It has no unit.

$$P.F = \frac{\text{True Power}}{\text{Apparent Power}}$$

$$P.F = \frac{VI\cos\phi}{VI}$$

$$\therefore \boxed{P.F = \cos\phi}$$

From Impedance triangle $R = Z\cos\phi$

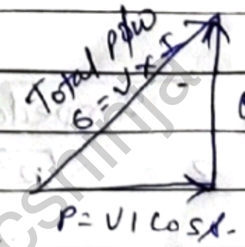
$$\therefore \boxed{\cos\phi = \frac{R}{Z}}$$

$$X_L = Z\sin\phi \qquad \boxed{\sin\phi = \frac{X_L}{Z}}$$

$$\tan\phi = X_L/R$$

* Power Triangle :

- Power triangle is a graphically represents the all Active, Reactive & Apparent P/W.

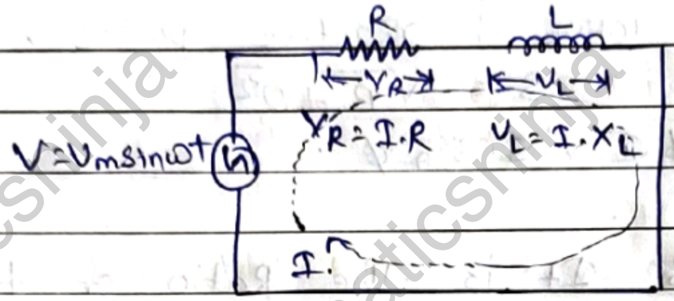


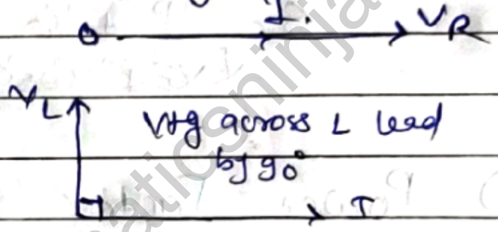
$$\phi = \tan^{-1}(X/R)$$

* AC Circuits with Series Elements :

1. Series R-L Circuit
2. Series R-C Circuit
3. Series R-L-C circuit

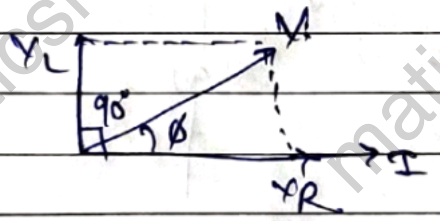
1) Series R-L circuit :



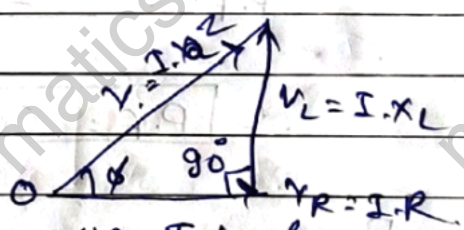


- In above fig. shows the circuit diagram of R-L series ckt.
- When the resistance & Inductance are connected in series with vtg or current source it is called as R-L Series ckt.

Voltage drop across R, $V_R = I.R$ } in series $V = V_R + V_L$
— " — L, $V_L = I.X_L$

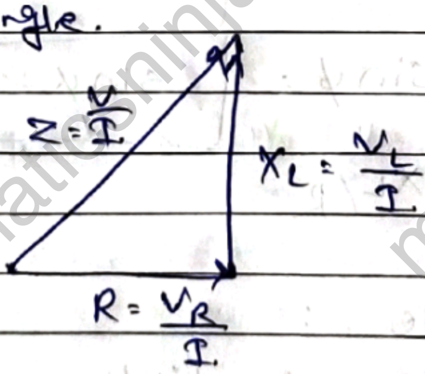


Phasor Diagram of L-R Ckt.



Vtg Triangle.

Impedance triangle.



Divided both side by I.

Impedance in polar form

$Z = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Q.1) A circuit takes a current of 8A at 100V, the current lagging by 30° behind the applied vtg. ~~across~~ Calculate, ① Impedance ② Resistance & ③ Inductance of the circuit with 50Hz frequency.

Soln :

Given, $I = 8A$, $V = 100V$, $\phi = 30°$

Find, $Z = ?$, $R = ?$, $L = ?$

$$Z = \frac{V}{I} = \frac{100}{8} = 12.5\,\Omega$$

$$\cos\phi = \frac{R}{Z} \quad \because (\text{from Impedance triangle})$$

$$R = Z\cos\phi$$
$$= 12.5\cos(30°)$$
$$R = 10.82\,\Omega$$

$X_L = 2\pi f L$ to find X_L

$$\sin\phi = \frac{X_L}{Z} \quad \because (\text{from Impedance triangle})$$

$$X_L = Z\sin\phi$$
$$= 12.5\sin(30°)$$
$$X_L = 6.25\,\Omega$$

$$X_L = 2\pi f L$$
$$L = \frac{X_L}{2\pi f} = \frac{6.25}{2\times 3.14\times 50} = 0.019\,H$$

Q.2) An ohmic resistance is connected in series with a coil across 230V, 50Hz supply. The current is 1.8 A and voltage across the resistance & coil are 80V & 170V respectively. Calculate Resistance & Inductance of the coil.

Soln :

Given, $V = 230V$, $I = 1.8A$, $F = 50Hz$

$V_R = 80V$, $V_L = 170V$

Find, $R = ?$

$L = ?$

$$R = \frac{V_R}{I} = \frac{80}{1.8} = 44.44\ \Omega$$

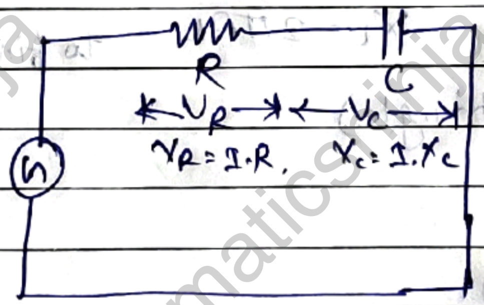
$$L = \frac{X_L}{2\pi F}$$

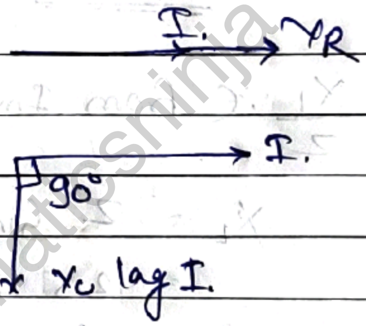
$$X_L = \sqrt{|Z|^2 - R^2}$$
$$= \sqrt{(127.78)^2 - (44.44)^2} = 119.77\ \Omega$$

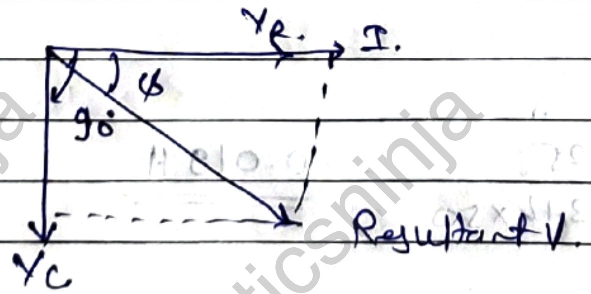
$$|Z| = \frac{V}{I} = \frac{230}{1.8} = 127.78\ \Omega$$

$$L = \frac{119.77}{2 \times 3.14 \times 50} = 0.38\ H$$

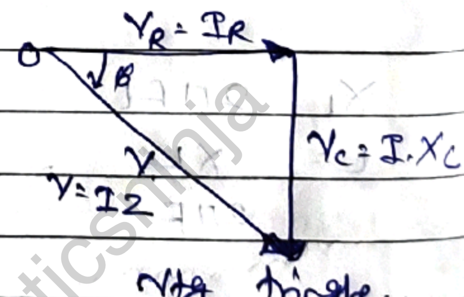
2) Series R-C Circuit :



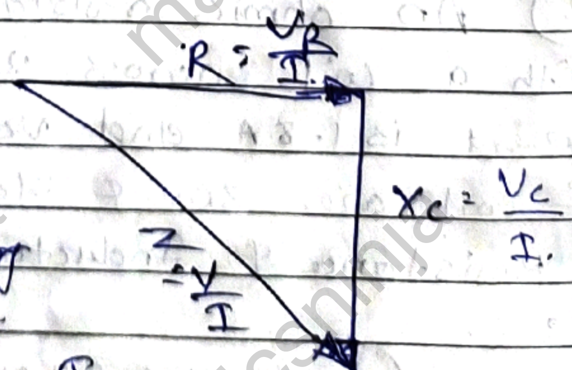




Phasor diagram



Vtg triangle.



Divided by I all sides.

Impedance triangle.

Impedance in series R-C sr ckt.

$Z = R - jX_C$ or $\cancel{|Z| = \sqrt{R^2 + X_C^2}}$

- In above fig. Shows the series connection with the Combination of R & C.
- A pure Capacitor (C) is connected in series with resistor, R & Vtg source of instantenous Vtg $V = V_m \sin \omega t$ is connected across the ~~seror~~ series combination of R & C.
- In case of series connection the applied Voltage V is equal to the phasor addition of V_R & V_C.

$\therefore$ Applied Vtg $\overline{V} = \overline{V_R} + \overline{V_c}$ (Phasor Addition)

Numerical :- **06M**

① A resistance of 100Ω & 50μF Capacitor are Connected in series across a 230V, 50 Hz Supply Find, ① Impedance ② The Current flowing through circuit ③ Voltage Across Resistance & Capacitance ④ Power factor & Power.

Soln :-

Given, $R = 100\Omega$, $C = 50\mu F = 50 \times 10^{-6} F$, $V = 230V$, $F = 50 Hz$

$$Z = \sqrt{R^2 + X_c^2}$$

$$X_c = \frac{1}{2\pi FC} = \frac{1}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}} = 63.69\ \Omega$$

$$Z = \sqrt{(100)^2 + (63.69)^2}$$
$$= 118.55\ \Omega$$

$$Z = \frac{V}{I} \quad \therefore I = \frac{V}{Z} = \frac{230}{118.55} = 1.94\ A.$$

$$V_R = I.R = 1.94 \times 100 = 194\ V$$

$$X_c = \frac{V_c}{I} \quad V_c = X_c . I = 63.69 \times 1.94 = 123.55\ V$$

$$P.F = \cos\phi = \frac{R}{Z} = \frac{100}{118.55} = 0.84$$

Power (P) = $V I \cos\phi$

$= 230 \times 1.94 \times 0.84$

$= 374.80$ Watt

$\boxed{\therefore Z = 118.55,\ V_R = 194V,\ V_C = 123.55,\ P.F = 0.8,\ I = 1.94,\ P = 374.80 \text{ Watt}}$

2) A resistance of 75Ω & $60\mu F$ are connected in series across 230V, 50 Hz supply find:
① X_C ② Current ③ P.f ④ Active P/W

Soln:

Given, $V = 230V$, $F = 50Hz$, $R = 75\Omega$, $C = 60\mu F$.

Find: X_C, I, P.F & P

$$X_C = \frac{1}{2\pi FC} = \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.05\Omega$$

$$I = \frac{V}{Z} = \frac{230}{91.87} = 2.503 A.$$

$$Z = \sqrt{(R)^2 + (X_C)^2}$$

$$= \sqrt{(75)^2 + (53.05)^2}$$

$$Z = 91.87\Omega$$

$$P.F \cos\phi = \frac{R}{Z} = \frac{75}{91.87} = 0.816$$

$$P = V I \cos\phi$$

$$= 230 \times 2.503 \times 0.816$$

$$P = 469.76 \text{ Watt}$$

$\boxed{\therefore I = 2.503 A,\ X_C = 53.05,\ P.F = 0.816,\ P = 469.76W}$

3) Find the expression for the current when vtg represent by $v = 283 \sin(100\pi t)$ is applyed to a coil having resistance is equal to 100Ω & capacitance $= 50\mu F$ also find power factor & frequency.

Soln: Given, $V_m = 283\,V$

$\omega = 100\pi$

$i = I_m \sin \omega t + \phi$

$R = 100\,\Omega$ & $C = 50 \times 10^{-6} F.$

$P.F = \cos\phi = \frac{R}{Z} = \frac{100}{118.55} = 0.84$ $\quad \phi = \cos^{-1}(0.84) = 32.85^\circ$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{(100)^2 + (63.69)^2} = 118.55$

$X_C = \frac{1}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}} = 63.69\,\Omega$

$\omega = 2\pi F$

$100\pi = 2\pi F.$

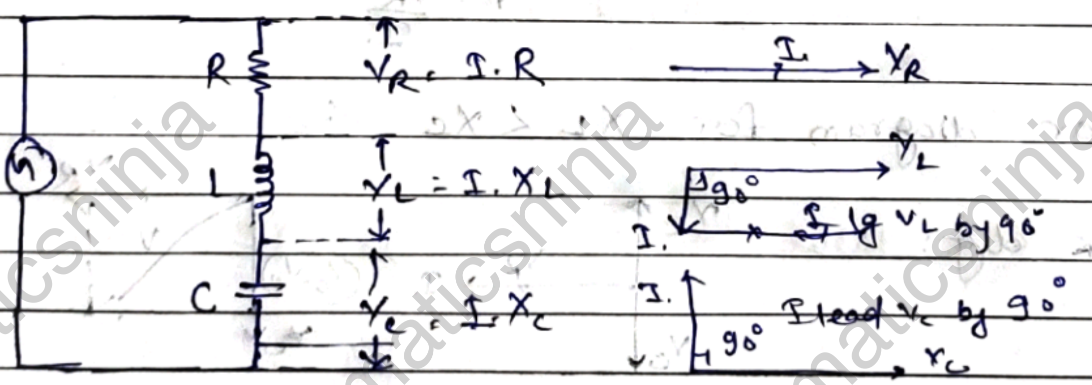
$F = \frac{100\pi}{2\pi} = 50\,Hz$

$I_m = \frac{V_m}{Z} = \frac{283}{118.55} = 2.38$

$i = I_m \sin \omega t + \phi$

$i = 2.38 \sin(100\pi t + 32.85)$

Series R-L-C circuit:



$\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$

$\vec{V} = \vec{I_R} . R + (\vec{I . X_L}) + \vec{I . X_C}$

$\vec{V} = \vec{I} (R + X_L + X_C)$

$\vec{I} = \frac{V}{R + j(X_L + X_C)}$

$I = \frac{V}{Z}$ (we got value of I)

Where, V_R, V_L & V_C are vtg across Resistance, Inductor & Capacitor respectively.

Phasor diagram :

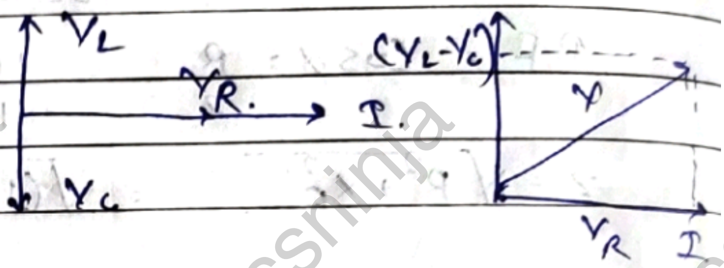
① Condition 1st : $X_L > X_C$

② Condition 2nd : $X_L < X_C$

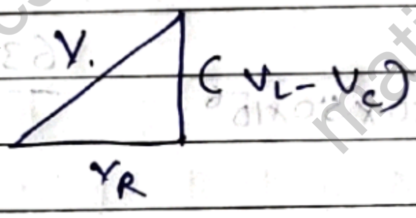
③ Condition 3rd : $X_L = X_C$

① Phasor diagram for $X_L > X_C$:

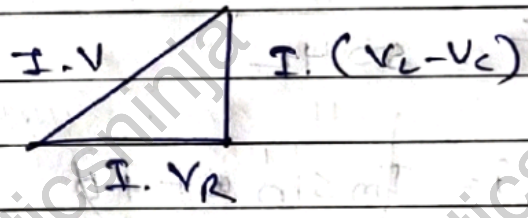
Phasor dig :



Vtg triangle :



Power triangle :

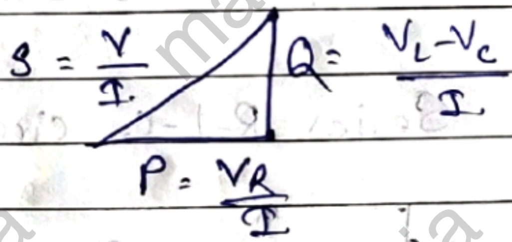


$S = VI$

$Q = I . (V_L - V_C)$

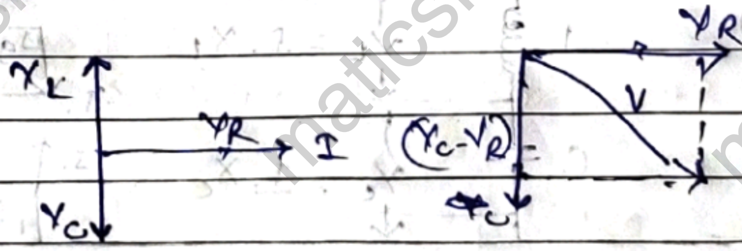
$P = I . V_R$

Impedance Triangle :

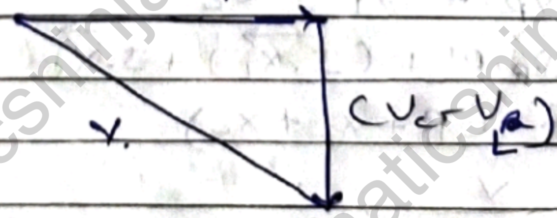


② Phasor diagram for $X_L < X_C$:

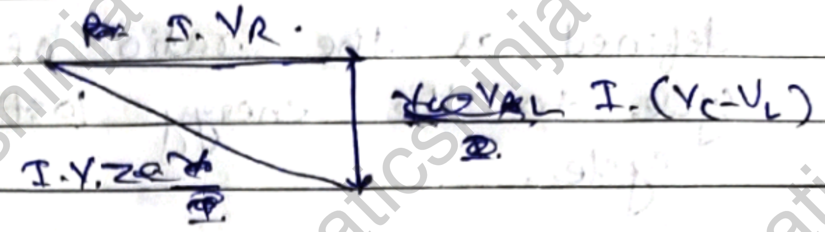
Phasor dig :



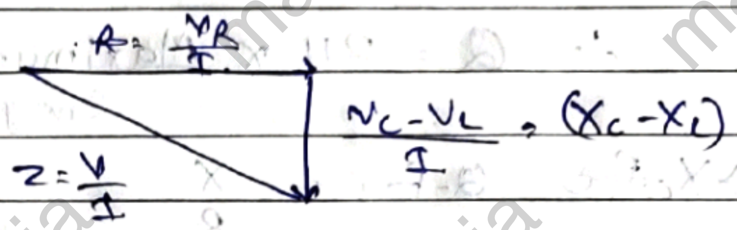
Vtg trigle :



P/ω ~~Vtg~~ triangle :



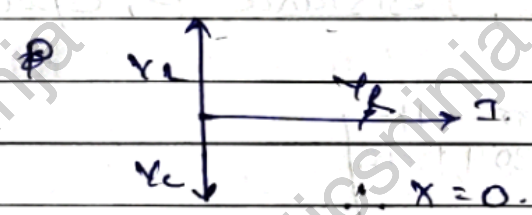
Impedance Triangle :



$$Z = R + jX$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

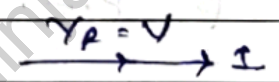
③ Phasor dig. for $X_L = X_C$



$V_L - V_C = 0$

$V_C - V_L = 0$

$\therefore X = 0.$



V & I are in phase.

$\therefore$ $\cancel{Z = \sqrt{R^2 + X^2}}$ $Z = R + jX$

$\cancel{Z = \sqrt{R^2}}$ $Z = R + j0$

$\therefore \boxed{Z = R}$

Power Factor :

Sr. No.	Case	Nature of RLC Ckt	Power Factor
1	$X_L > X_C$	R - L series	Lagging & less than 1
2	$X_C > X_L$	R - C Series	Leading & less than 1
3	$X_L = X_C$	Purely Resistive	1

Q. Factor : The Q (Quality) Factor is defined as the ratio of energy stored per cycle to the energy lost (or dissipated) per cycle.

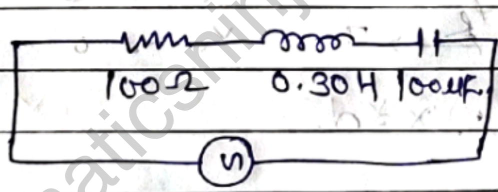
$$\therefore \quad Q = 2\pi \times \frac{\text{Maximum Energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

$X_L > X_C$ for $\quad Q.F = \frac{X}{R} = \frac{X_L - X_C}{R}.$

$X_C > X_L \quad Q.F = \frac{X}{R} = \frac{X_C - X_L}{R}$

Numericals :

1) From Given ckt find Impedance, Current, Vtg across Inductance, Resistance &, Capacitance & P.F.



$V = 200\sin(100\pi t)$

Sol^n : Given, $R = 100\Omega$, $L = 0.30\,H$, $C = 100\,\mu F$, $V_m = 200V$, $\omega = 100\pi$

Find : Z, I, V_L, V_R, V_C & P.F.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = 2\pi F L = 2 \times 3.14 \times 50 \times 0.30 = 94.2\,\Omega$$

$$X_C = \frac{1}{2\pi F C} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}} = 31.8\,\Omega$$

$$Z = \sqrt{100^2 + (94.2 - 31.8)^2} = 117.87\,\Omega$$

$$I = \frac{V}{Z} = \frac{200}{117.87} = 1.69\,A.$$

$V_L = I \cdot X_L$
$= 1.69 \times 94.2$
$= 159.19\ V$

$V_R = I \cdot R = 1.69 \times 100 = 169\ V$

$V_C = I \cdot X_C = 1.69 \times 31.8 = 53.74\ V$

$P.F = \cos\phi = \frac{R}{Z} = \frac{100}{117.87} = 0.84$

2) A series circuit having pure resistance of 40Ω & inductance of 50.7 mH & capacitor is connected across 400V, 50 Hz, AC supply. This R-L-C Combination draws a current of 10 A. Calculate:
① P.F of the circuit ② Capacitor value.

Soln:

Given, $R = 40\Omega$, $L = 50.7\ mH = 50.7 \times 10^{-3}\ H$, $V = 400V$, $f = 50$, $I = 10A$.

Find: P.F & C

$P.F = \cos\phi = \frac{R}{Z} = \frac{40}{40} = 1$ $V = \frac{I}{Z}$ $Z =$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (15.72 - 15.72)^2} = 40\Omega$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 50.7 \times 10^{-3} = 15.972\ \Omega$

According to 3rd condition $X_L = X_C$

$\therefore X_C = 15.72\ \Omega$

$X_C = \frac{1}{2\pi f C}$

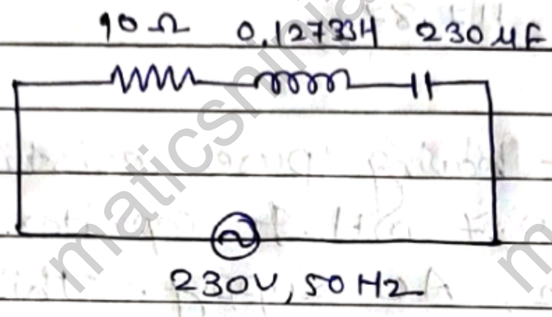
$C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times 3.14 \times 50 \times 15.72} = 0.000202$ or 2.02×10^{-4} or $202 \times 10^{-6}\ F$
$202\ \mu F$

$\boxed{I = \frac{V}{Z} \quad \therefore Z = \frac{V}{I} = \frac{400}{10} = 40\Omega}$

08 Marks

3) A coil has resistance of 10 ohm & Inductance of 0.12733 H. This coil is connected in series with a capacitor of 230 µF across the source of supply of 230V, 50 Hz Find :
① X_L ② X_C ③ V/g across coil & ⑥ capacitor ④ P.F ⑤ Z ⑥ Current ⑦ Angle of phase displacement betⁿ V/g & current.

Solⁿ :



Given From ckt dig.
R, L, C & V, F.

① $X_L = 2\pi FL = 2\times 3.14 \times 50 \times 0.12733 = 40\,\Omega$

② $X_C = \dfrac{1}{2\pi FC} = \dfrac{1}{2\times 3.14\times 50\times 230\times 10^{-6}} = 13.83\,\Omega$

③ $V_L = I \cdot X_L = 8.21 \times 40 = 328\ V$

④ $I = \dfrac{V}{Z} = \dfrac{230}{28} = 8.21\ A.$

⑤ $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (40 - 13.83)^2} = 28\,\Omega$

⑥ $V_C = I \cdot X_C = 113.54\ V$

⑦ $P.F = \cos\phi = \dfrac{R}{Z} = \dfrac{10}{28} = 0.35$

⑧ $\phi = \tan^{-1}\left(\dfrac{X}{R}\right) = \tan^{-1}(26.17/10) = 69.08^\circ$

$\therefore X = X_L - X_C$
$X = 26.17$

* Resonance frequency : The frequency at which inductive Reactance becomes equal to the Capacitive Reactance is called as resonance frequency

Expression for Resonance frequency

$$X_L = X_C$$

$$2\pi F_r L = \frac{1}{2\pi F_r C}$$

$$F_r \times F_r^2 = \frac{1}{2\pi \times 2\pi L \cdot C}$$

$$F_r^2 = \frac{1}{4\pi^2 L \cdot C}$$

$$F_r = \sqrt{\frac{1}{4\pi^2 L \cdot C}}$$

$$\boxed{F_r = \frac{1}{2\pi\sqrt{L \cdot C}}}$$ ∴ (formula for Resonance frequency)

At Resonant $|Z| = R$

Q - Factor of R-L-C series circuit :

- The Q-Factor of series resonant circuit is also defined as the Voltage magnification provided by the circuit at its resonant frequency.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$V_L = Q \times V$$

$$Q = \frac{V_L}{V}$$

$$= \frac{\cancel{I} \cdot X_L}{\cancel{I} \cdot R}$$

$$= \frac{X_L}{R}$$

$$= \frac{2\pi F_r L}{R}$$

$$= \frac{\cancel{2\pi} \frac{1}{\cancel{2\pi}\sqrt{LC}} L}{R}$$

$$= \frac{\frac{1}{LC} \times L^{\cancel{2}}}{R^2}$$

Taking square

$$\frac{\left(\frac{1}{\sqrt{LC}}\right)^2 L^2}{R^2}$$

Taking square Root

$$Q = \sqrt{\frac{1}{R^2} \times \frac{L}{C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

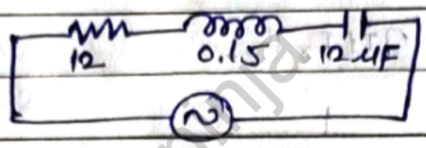
∴ Formula for Q

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Numerical on Resonante Frequeny :

1> A circuit consisting of a coil of Resistance $12\,\Omega$ & Inductance 0.15 H in series with capacitance of $12\,\mu F$ is connected to a variable frequency supply which has a constant vtg 240V Calculate : ① Resonant frequency ② Current in cir.

Soln :



240v,s

Given,

$$R = 12\,\Omega \;,\; L = 0.15 \;,\; C = 12\,\mu F$$

Find : F_r & I

$$F_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.15 \times 12 \times 10^{-6}}} = 118.63\,Hz$$

~~At Resonant |Z| = R~~ $\therefore$ ~~Z = 12~~

$$I = \frac{V}{R} = \frac{240}{12} = 20\,A.$$

2> An AC series circuit having $R = 10\,\Omega$, $L = 0.1$ H & $C = 100\,\mu F$ is connected to 230 V, 50 Hz supply find ① Resonance frequency ② Quality factor ③ I

Sol^n :

$$F_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 100 \times 10^{-6}}} = 50.35\,Hz$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.1}{100 \times 10^{-6}}} = 3.16$$

$$I = \frac{V}{R/Z} = \frac{230}{10} = 23\,A.$$