2.1

Standard Boolean representation :

* Classification of digital circuits :

1) Combinational Logic Circuits.
2) Sequential Logic Circuits

1) Combinational logic circuits :

The combinational circuit the output of the circuit depends only on the inputs present. Therefore, combinational ckt do not need memory.
e.g. Adders, subtractor, code converter etc.

2) Sequential Logic Circuits :

The output of sequential circuit is depend on the present input as well as past input output, so sequential circuit requires memory to store past value.
e.g flip-flop, shift register, counters etc.

* Types of Logic Expression :

1) SOP (sum of product)
2) POS (product of sum).

1) SOP form :

In this form the individual term is of product and we are performing sum of the individual terms.

e.g. $Y = (AB) + (AC)$

2) POS form :

In this form the individual term is of sum and we are performing product of the individual terms.

e.g. $Y = (A+C) \cdot (A+B)$

* Canonical and standard SOP form :

Steps :

1) find the missing literal form from each individual term.
2) Perform ORing the missing literal with its complement and ANDing that term with previous input term.
3) Simplify the obtain expression using boolean laws.

Examples :

Q. Obtained standard form of given logic expression

1) $Y = A+B$

→ ① find missing literal

$Y = \underset{\uparrow \atop B}{A} + \underset{\uparrow \atop A}{B}$

② ORing the missing literal with its complement and perform ANDing.

$$Y = A\cdot(B+\bar{B}) + B(A+\bar{A})$$
$$= AB + A\bar{B} + BA + B\bar{A}$$
$$= AB + A\bar{B} + AB + B\bar{A}$$

$$\boxed{Y = AB + A\bar{B} + \bar{A}B}$$

② $Y = AB + A\bar{C} + BC$

→

step① find missing literal

$$Y = AB + A\bar{C} + BC$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$C \quad B \quad A$$

② ORing the missing literal with its complement and perform ANDing.

$$Y = ABC(C+\bar{C}) + A\bar{C}(B+\bar{B}) + BC(A+\bar{A})$$
$$= ABC + AB\bar{C} + A\bar{C}B + A\bar{C}\bar{B} + BCA + BC\bar{A}$$
$$= \underline{ABC} + \underline{AB\bar{C}} + \underline{AB\bar{C}} + A\bar{B}\bar{C} + \underline{ABC} + \bar{A}BC$$

$$\boxed{Y = ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC}$$

3) $Y = \bar{A} + B\bar{C}\bar{D}$

→

① find missing literal

$$Y = \bar{A} + B\bar{C}\bar{D}$$
$$\downarrow \quad \downarrow$$
$$BCD \quad A$$

2) ORing the missing literal with its complement and perform ANDing.

$$Y = A(B+\bar{B}) \cdot (C+\bar{C})(D+\bar{D}) + (A+\bar{A}) \cdot B\bar{C}\bar{D}$$
$$= (AB + A\bar{B}) \cdot (CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + AB\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D}$$
$$= ABCD + ABC\bar{D} + AB\bar{C}D + \underline{AB\bar{C}\bar{D}} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \underline{AB\bar{C}\bar{D}} + \bar{A}B\bar{C}\bar{D}$$

$$\therefore \boxed{Y = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D}}$$

* **Minterm** : The each individual term in standard SOP format is called as minterm. It is represented by m.

e.g. $Y = \underbrace{ABC}_{\text{minterm}} + \underbrace{\bar{A}BC}_{\text{minterm}}$

Truth table for two i/p variables :

inputs		Minterm's (m)	m
A	B		
0	0	$\bar{A}\bar{B}$	m_0
0	1	$\bar{A}B$	m_1
1	0	$A\bar{B}$	m_2
1	1	AB	m_3

Truth table for 3 input variables :

inputs			minterm	m
A	B	C		
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

* Representation of SOP form in minterm :

* Obtain the equation of minterm form

1) $Y = ABC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$

$Y = m_7 + m_2 + m_1 + m_6$

$Y = \Sigma m(7, 2, 1, 6)$

$$\boxed{Y = \Sigma m(1, 2, 6, 7)}$$

2) $Y = \bar{A}B + A\bar{B}$

$Y = m_1 + m_2$

$$\boxed{Y = \Sigma m(1, 2)}$$

3) $Y = ABCD + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + AB\bar{C}\bar{D}$

$Y = m_{15} + m_5 + m_3 + m_{12}$

$Y = \Sigma m(15, 5, 3, 12)$

$$\boxed{Y = \Sigma m(3, 5, 12, 15)}$$

* Representation of SOP form from truth table

Step ① Consider the combination of inputs having output or $Y = 1$

Step ② Write the individual product terms having $Y=1$ and perform sum of individual term.

Step ③ Simplify.

Write canonical SOP form from truth table

Q1)

A	B	Y	
0	0	0	
0	1	1	— Y_1
1	0	1	— Y_2
1	1	0	

→ ① Consider combinations where $Y = 1$

$Y_1 = 1 \quad \therefore \quad A=0,\ B=1$

$Y_2 = 1 \quad \therefore \quad A=1,\ B=0$

② Write SOP form,

$$Y = Y_1 + Y_2$$

$$Y = \overline{A}B + A\overline{B}$$

$$\therefore \boxed{Y = \overline{A}B + A\overline{B}}$$

Scanned by CamScanner

(2)

A	B	C	Y	
0	0	0	0	
0	0	1	1	Y_1
0	1	0	0	
0	1	1	0	
1	0	0	1	Y_2
1	0	1	1	Y_3
1	1	0	0	
1	1	1	1	Y_4

→ Consider combination $Y = 1$

$Y_1 = 1$, $\therefore A=0,\ B=0,\ C=1$, $\bar{A}\bar{B}C$

$Y_2 = 1$, $\therefore A=1,\ A=0,\ C=0$ $\quad A\bar{B}\bar{C}$

$Y_3 = 1$, $\therefore A=1,\ B=0,\ C=1$ $\quad A\bar{B}C$

$Y_4 = 1$, $\therefore A=1,\ B=1,\ C=1$ $\quad ABC$

$$Y = Y_1 + Y_2 + Y_2 + Y_4$$

$$Y = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$Y = m_1 + m_4 + m_5 + m_7$$

$$\boxed{Y = \Sigma m(1,4,5,7)}$$

(3)

A	B	C	Y	
0	0	0	1	Y_1
0	0	1	0	
0	1	0	1	Y_2
0	1	1	0	
1	0	0	1	Y_3
1	0	1	1	Y_4
1	1	0	0	
1	1	1	0	

→ Consider Combination

$Y_1 = 1 \quad \therefore \quad A=0, \; B=0, \; C=0 \quad , \quad \bar{A}\bar{B}\bar{C}$
$Y_2 = 1 \quad \therefore \quad A=0, \; B=1, \; C=0 \quad , \quad \bar{A}B\bar{C}$
$Y_3 = 1 \quad \therefore \quad A=1, \; B=0, \; C=0 \quad , \quad A\bar{B}\bar{C}$
$Y_4 = 1 \quad \therefore \quad A=1, \; B=0, \; C=1 \quad , \quad A\bar{B}C$

write SOP.

$$Y = Y_1 + Y_2 + Y_3 + Y_4$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$$
$$Y = m_0 + m_2 + m_4 + m_5$$

$$\boxed{Y = \Sigma m\,(0, 2, 4, 5)}$$

* Canonical or standard POS form :

Steps:

① find the missing literal from each individual term

② Perform ANDing the missing literal with its complement and ORing that with previous term.

③ Simplify.

Q. find standard or canonical form of given boolean expression.

1) $Y = (A+B)(A+C)$

→

① find missing literal

$$Y = \underset{\downarrow \atop C}{(A+B)}\ \underset{\downarrow \atop B}{(A+C)}$$

② POS form is,

$Y = (A+B)+(C\cdot\bar{C})\cdot(A+C)+(B\bar{B})$

$Y = \underline{(A+B+C)}\cdot(A+B+\bar{C})\cdot\underline{(A+B+C)}\cdot(A+\bar{B}+C)$

$\therefore \boxed{Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)}$

2) $Y = (A+C)\cdot(B+C)\cdot(\bar{A}+B)$

→ ① find missing literal

$$Y = \underset{\downarrow \atop B}{(A+C)}\cdot\underset{\downarrow \atop A}{(B+C)}\cdot\underset{\downarrow \atop C}{(\bar{A}+B)}$$

② POS form is,

$Y = (A+C)+(B\bar{B})\cdot(B+C)+(A\cdot\bar{A})\cdot(\bar{A}+B)+(C\bar{C})$

$Y = \underline{(A+B+C)}\cdot(A+\bar{B}+C)\cdot\underline{(A+B+C)}\cdot\underline{(\bar{A}+B+C)}\cdot$
$\underline{\underline{(\bar{A}+B+C)}}\cdot(\bar{A}+B+\bar{C})$

$Y = (A+B+C)\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C})$.

3) $Y = (\bar{A}+B)\cdot(B+\bar{C})\cdot(\bar{A}+\bar{C})$

→ ① find missing literal

$$Y = \underset{\downarrow \atop C}{(\bar{A}+B)}\cdot\underset{\downarrow \atop A}{(B+\bar{C})}\cdot\underset{\downarrow \atop B}{(\bar{A}+\bar{C})}$$

Scanned by CamScanner

③ POS form is,

$$Y = (\bar A + B) + (C\cdot\bar C)\cdot(B+\bar C) + (A\bar A)\cdot(\bar A+\bar C) + (B\bar B)$$

$$Y = (\bar A+B+C)\cdot(\bar A+B+\bar C)\cdot(A+B+\bar C)\cdot(\bar A+B+\bar C)$$
$$(\bar A+B+\bar C)\cdot(\bar A+\bar B+\bar C)$$

$$Y = (\bar A+B+C)\cdot(\bar A+B+C)\cdot(A+B+\bar C)\cdot(\bar A+B+\bar C)(\bar A+\bar B+\bar C)$$

4) $Y = A\cdot(B+C+D)$

→

$Y = A\cdot(B+C+D)$

① find missing literal,

$$Y = \underset{\substack{\downarrow \\ BCD}}{A} \cdot \underset{\substack{\downarrow \\ A}}{(B+C+D)}$$

② POS form is,

$$Y = A+(B\cdot\bar B)+(C\cdot\bar C)+(D\cdot\bar D)\cdot(B+C+D)+(A\bar A)$$

$$Y = A+(B\cdot\bar B)+(C+D\cdot C+\bar D\cdot \bar C D\cdot \bar C\bar D)\cdot(A+B+C+D)$$
$$\cdot(\bar A+B+C+D)$$

$$Y = (A+B)\cdot(A+\bar B)+[(C+D)\cdot(C+\bar D)(\bar C+D)\cdot(\bar C+\bar D)]$$
$$\cdot(A+B+C+D)\cdot(\bar A+B+C+D)$$

$$Y = (A+B+C+D)\cdot(A+B+C+\bar D)\cdot(A+B+\bar C+D)\cdot(A+B+\bar C+\bar D)$$
$$(A+\bar B+C+D)\cdot(A+\bar B+C+\bar D)\cdot(A+\bar B+\bar C+D)\cdot(A+\bar B+\bar C+\bar D)$$
$$(\bar A+B+C+D)$$

* **Maxterm :** The individual term in the standard POS form is called as maxterm.
It is represented by 'M'.

e.g. $Y = (A+B)\cdot(A+\bar{B})$

$$\underbrace{(A+B)}_{\text{maxterm}} \cdot \underbrace{(A+\bar{B})}_{\text{maxterm}}$$

Truth Table for two input variables :

inputs		Maxterm	M
A	B		
0	0	$A+B$	M_0
0	1	$A+\bar{B}$	M_1
1	0	$\bar{A}+B$	M_2
1	1	$\bar{A}+\bar{B}$	M_3

Truth Table for three input variables :

inputs			maxterm	M
A	B	C		
0	0	0	$A+B+C$	M_0
0	0	1	$A+B+\bar{C}$	M_1
0	1	0	$A+\bar{B}+C$	M_2
0	1	1	$A+\bar{B}+\bar{C}$	M_3
1	0	0	$\bar{A}+B+C$	M_4
1	0	1	$\bar{A}+B+\bar{C}$	M_5
1	1	0	$\bar{A}+\bar{B}+C$	M_6
1	1	1	$\bar{A}+\bar{B}+\bar{C}$	M_7

* Representation of POS form in maxterm :

* Obtain the equation of maxterm form,

1) $Y = (A+B)\cdot(\bar{A}+\bar{B})$

$Y = M_0 \cdot M_3$

$\boxed{Y = \Pi M\,(0,3)}$

2) $Y = (A+B+C)\cdot(A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+\bar{B}+\bar{C})$

$Y = M_0 \cdot M_2 \cdot M_3 \cdot M_7$

$\boxed{Y = \Pi M\,(0,2,3,7)}$

3) $Y = (A+B+C+D)\cdot(A+\bar{B}+\bar{C}+D)\cdot(A+\bar{B}+\bar{C}+\bar{D})\cdot$
$(\bar{A}+\bar{B}+\bar{C}+\bar{D})\cdot(\bar{A}+B+C+D)$

$Y = M_0 \cdot M_6 \cdot M_7 \cdot M_{15} \cdot M_8$

$\boxed{Y = \Pi M\,(0,6,7,8,15)}$

Date
Page

* Representation of POS form from truth table :

Step ① consider the combination of inputs having output or $Y=0$.

Step ② write the individual sum term's having $Y=0$ & perform product of individual term

Step ③ simplify.

* Write canonical SOP form from truth table :

Q1)

A	B	Y	
0	0	0	Y_1
0	1	1	
1	0	1	
1	1	0	Y_2

→ ① Consider combination's where $Y=0$,

$Y_1 = 0$, ∴ $A=0$, $B=0$ ∴ $A+B$

$Y_2 = 0$, ∴ $A=1$, $B=1$, ∴ $\bar{A}+\bar{B}$

② write POS form,

$$Y = Y1 \cdot Y2$$

$$\boxed{Y = (A+B)\cdot(\bar{A}+\bar{B})}$$

$$Y = M_0 \cdot M_3$$

$$\boxed{Y = \Pi M(0,3)}$$

Q2)

A	B	C	Y	
0	0	0	1	
0	0	1	1	
0	1	0	0	Y_1
0	1	1	0	Y_2
1	0	0	1	
1	0	1	1	
1	1	0	0	Y_3
1	1	1	0	Y_4

→

① Consider combination where $Y = 0$

$Y_1 = 0$, $A=0, B=1, C=0,$ ~~B=0~~ , $A+\overline{B}+C$

$Y_2 = 0$, $A=0, B=1, C=1$, $A+\overline{B}+\overline{C}$

$Y_3 = 0$, $A=1, B=1, C=0$, $\overline{A}+\overline{B}+C$

$Y_4 = 0$, $A=1, B=1, C=1$, $\overline{A}+\overline{B}+\overline{C}$

② write POS form,

$$Y = Y_1 \cdot Y_2 \cdot Y_3 \cdot Y_4$$

$$Y = (A+\overline{B}+C)\cdot(A+\overline{B}+\overline{C})\cdot(\overline{A}+\overline{B}+C)\cdot(\overline{A}+\overline{B}+\overline{C})$$

K-map Simplification :

K-map Structure :

* 2 variable K-map :

no. of i/p's = 2
no. of cell's = $2^2 = 4$

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	$\bar{A}\bar{B}$	$\bar{A}B$
A 1	$A\bar{B}$	AB

or

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	m_0	m_1
A 1	m_2	m_3

* 3 variable K-map :

no. of i/ps = 3
no. of cell's = $2^3 = 8$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
A 1	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

or

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	m_0	m_1	m_3	m_2
A 1	m_4	m_5	m_7	m_6

* 4 variable K-map :

no. of i/ps = 4
no. of cell's = $2^4 = 16$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$ 01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
AB 11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
$A\bar{B}$ 10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

or

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	m_0	m_1	m_3	m_2
$\bar{A}B$ 01	m_4	m_5	m_7	m_6
AB 11	m_{12}	m_{13}	m_{15}	m_{14}
$A\bar{B}$ 10	m_8	m_9	m_{11}	m_{10}

* Plotting a Boolean Expression :

① Consider the expression -

$$Y = \bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}$$

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

K-map :-

A \ BC		$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$	0	1 $_0$	1 $_1$	1 $_3$	0 $_2$
A	1	0 $_4$	0 $_5$	0 $_7$	0 $_6$

② Plot the truth table on K-map -

A	B	Y	
0	0	0	
0	1	0	
1	0	0	
1	1	1	→ AB

A \ B		$\bar{B}$ 0	B 1
$\bar{A}$	0	0	0
A	1	0	1

(Q3)

A	B	C	D	Y	
0	0	0	0	0	
0	0	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}D = m_1$
0	0	1	0	0	
0	0	1	1	1	$\rightarrow \bar{A}\bar{B}CD = m_3$
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	$\rightarrow \bar{A}BC\bar{D} = m_6$
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	$\rightarrow A\bar{B}\bar{C}D = m_9$
1	0	1	0	0	
1	0	1	1	1	$\rightarrow A\bar{B}CD = m_{11}$
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	$\rightarrow ABCD = m_{15}$

K-map :-

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	0	0	1
AB 11	0	0	1	0
$A\bar{B}$ 10	0	1	1	0

* Grouping of 1's in K-map :

i) The pair : A pair is a group of two adjacent 1's in a K-map. A pair leads to elimination of one variable in SOP form.

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1	0
A 1	0	0	0	0

$$Y = \bar{A}\bar{B}C + \bar{A}BC$$
$$= \bar{A}C(\bar{B}+B)$$
$$= \bar{A}C\cdot(1)$$
$$\boxed{Y = \bar{A}C}$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	1	0
A 1	0	0	1	0

$$Y = \bar{A}BC + ABC$$
$$= BC(\bar{A}+A)$$
$$\boxed{Y = BC}$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	0	0	1
A 1	0	0	0	0

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}$$
$$= \bar{A}\bar{C}(\bar{B}+B)$$
$$Y = \bar{A}\bar{C}$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$	1	1	1	1
A	1	1	1	1

$$\boxed{Y = 1}$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01	1			1
AB 11				
$A\bar{B}$ 10				

$$Y = \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D}$$
$$= \bar{A}B\bar{D}\,(\bar{C}+C)$$

$$\boxed{Y = \bar{A}B\bar{D}}$$

ii) The Quad : A group of four adjacent 1's on k-map is known as quad. A quad eliminates two variables.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01	1	1	1	1
AB 11				
$A\bar{B}$ 10				

$$\boxed{Y = \bar{A}B}$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00			1	
$\bar{A}B$ 01			1	
AB 11			1	
$A\bar{B}$ 10			1	

$$\boxed{Y = CD}$$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00			1	1
$\bar{A}B$ 01				
AB 11				
$A\bar{B}$ 10			1	1

$$\boxed{Y = \bar{B}C}$$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01				
AB 11	1			1
$A\bar{B}$ 10	1			1

$$\boxed{Y = A\bar{D}}$$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1			1
$\bar{A}B$ 01				
AB 11				
$A\bar{B}$ 10	1			1

$$\boxed{Y = \bar{B}\bar{D}}$$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01		1	1	
AB 11		1	1	
$A\bar{B}$ 10				

$$\boxed{Y = BD}$$

(iii) **The Octet :** A Octet is a group of eight adjacent 1's. It leads elimination of three variables.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10				

$\boxed{Y = B}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1		
$\bar{A}B$ 01	1	1		
AB 11	1	1		
$A\bar{B}$ 10	1	1		

$\boxed{Y = \bar{C}}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1			1
$\bar{A}B$ 01	1			1
AB 11	1			1
$A\bar{B}$ 10	1			1

$\boxed{Y = \bar{D}}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01				
AB 11				
$A\bar{B}$ 10	1	1	1	1

$\boxed{Y = \bar{B}}$

* Rules for k-map Simplification :

1. A k-map may have more than one pair, quad or octet.
2. The 1's left without groups must also be encircled.
3. While grouping, it may be remembered that overlapping of groups is allowed.
4. Redundancy is not allowed i.e. a group whose all 1's are overlapped by other groups.
5. For simplifying SOP equations, groups should include only cells containing 1's and for POS equations groups should contain only 0's.
6. Groups can be horizontal or vertical but not diagonal
7. Group should be as large as possible.
8. Groups can be wrapped around the table.

* Simplification of SOP expression using k-map -

1. Draw the k-map. Depending on the number of variables, an equation with n variables, will require a k-map with 2^n cells.
   e.g. 3 variables, k-map has $2^3 = 8$ cells.
   4 variables, k-map has $2^4 = 16$ cells.
2. Plot the k-map from given eqn or truth table. for SOP put 1. & for POS put 0.
3. Group of 1's: Try to form a larger group to eliminate max. variables
4. Remaining 1 must be encircle, & consider the minterms to find simplified o/p expression.

* Minimize the given logical expression using k-map technique.

① $Y = A\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}\bar{B}\bar{C}$

→ no. of variables = 3
no. of cell's = $2^3 = 8$.

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	0	0
A 1	1	1	0	0

quad 1

$\therefore \boxed{Y = \bar{B}}$

② $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$

no. of variables = 3
no. of cell's = $2^3 = 8$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	0	1	1
A 1	0	1	0	0

encircle ①
pair 2
pair 1

$Y =$ pair 1 + pair 2 + encircle ①

$$\boxed{Y = \bar{A}B + \bar{A}\bar{C} + A\bar{B}C}$$

③ Draw the k-map & find the minimized logical expression.

$$Y = \Sigma m\,(0,1,2,4,12)$$

→

$$Y = \Sigma m\,(0,1,2,4,12)$$

$$Y = m_0 + m_1 + m_2 + m_4 + m_{12}$$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	1_0	1_1	0_3	1_2
$\bar A B$ 01	1_4	0_5	0_7	0_6
AB 11	1_{12}	0_{13}	0_{15}	0_{14}
$A\bar B$ 10	0_8	0_9	0_{11}	0_{10}

pair 2

pair 1

pair 3

$$Y = \text{pair1} + \text{pair2} + \text{pair3}$$

$$Y = \bar A\bar B\bar C + \bar A\bar B\bar D + B\bar C\bar D$$

Q4) For the logical expression given below, draw the k-map and obtain the simplified logical expression $Y = \Sigma m\,(1,5,7,9,11,13,15)$.

→

$$Y = \Sigma m\,(1,5,7,9,11,13,15)$$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	0	1	0	0
$\bar A B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar B$ 10	0	1	1	0

quad 3

quad 1 quad 2

$$Y = \text{quad}_1 + \text{quad}_2 + \text{quad}_3$$

$$\therefore \boxed{Y = \bar{C}D + AD + BD}$$

5) Reduce the following eqn by K-map.

$$Y = \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}\bar{C}D + AB\bar{C}D + AB\bar{C}\bar{D} + ABCD$$

→

$$Y = \bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D + AB\bar{C}D + AB\bar{C}\bar{D} + ABCD$$

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	1	0	0
$\bar{A}B$	01	0	1	0	0
AB	11	1	1	1	0
$A\bar{B}$	10	0	1	0	0

quad1 pair1 pair2

$$Y = \text{quad}_1 + \text{pair}_1 + \text{pair}_2$$

$$\therefore \boxed{Y = \bar{C}D + AB\bar{C} + ABD}$$

6) Find out the minimal expression for the function.

$$Y = (A, B, C, D) = \Sigma m(0, 2, 4, 6, 8, 9, 12)$$

→

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	1	0	0	1
$\bar{A}B$	01	1	0	0	1
AB	11	1	0	0	0
$A\bar{B}$	10	1	1	0	0

quad2 quad1 pair1

$Y = \text{quad}_1 + \text{quad}_2 + \text{pair}_1$

$$\therefore \boxed{Y = \bar{C}\bar{D} + \bar{A}\bar{D} + A\bar{B}\bar{C}}$$

⑦ Implement the logic function specified by the truth table, using k-map.

→

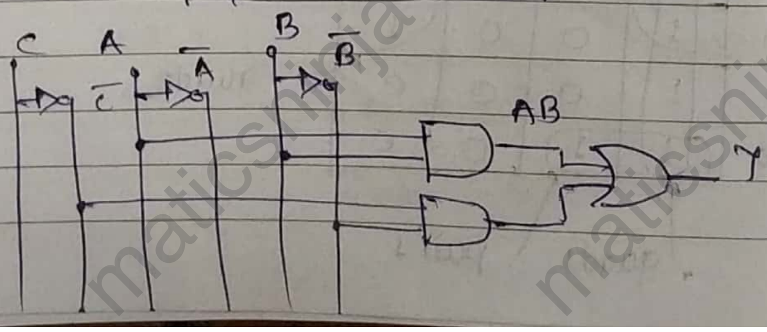
Input			Output
A	B	C	Y
0	0	0	1 → m_0
0	0	1	0
0	1	0	0
0	1	1	0
~~0~~1	0	0	1 → m_4
~~0~~1	0	1	0
~~0~~1	1	0	1 → m_6
~~0~~1	1	1	1 → m_7

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	0	0	0
A 1	1	0	1	1

pair1   pair2

$Y = \text{pair}_1 + \text{pair}_2$

$$\therefore \boxed{Y = \bar{B}\bar{C} + AB}$$



⑧ Design the logic circuit which receive four bit binary number gives out an o/p whatever the number is divisible by 4 or 5.

A	B	C	D	Y	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	1	→ m_4
0	1	0	1	1	→ m_5
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	1	→ m_8
1	0	0	1	0	
1	0	1	0	1	→ m_{10}
1	0	1	1	0	
1	1	0	0	1	→ m_{12}
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	→ m_{15}

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	0	0
AB 11	1	0	1	
$A\bar{B}$ 10	1	0	0	1

encircle ①

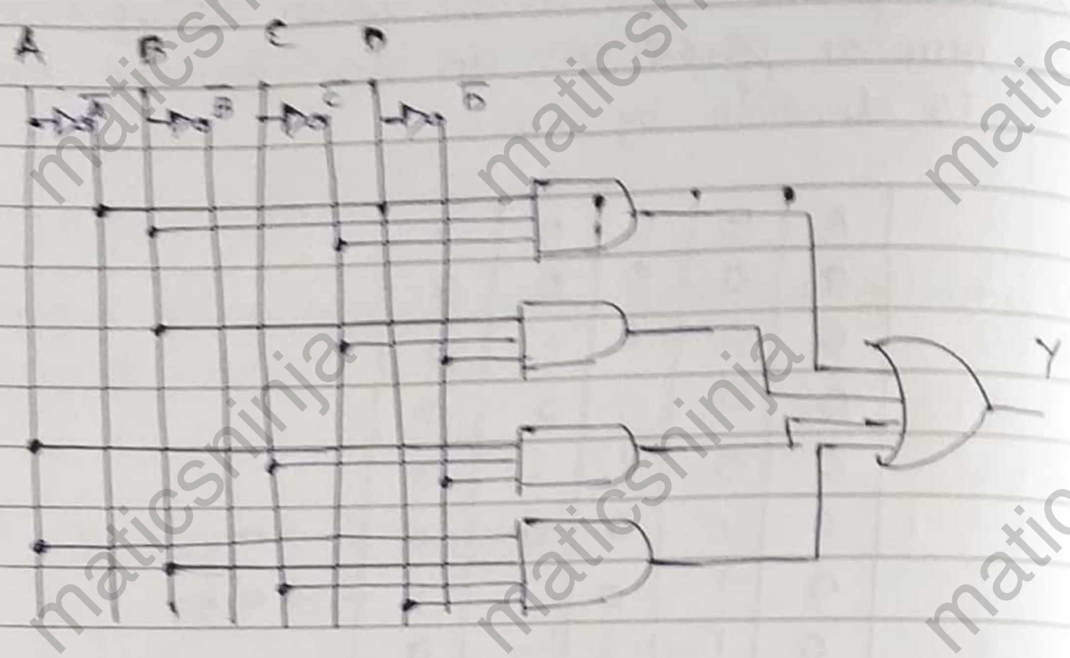
pair1

pair2

pair3

$$Y = \text{pair}_1 + \text{pair}_2 + \text{pair}_3 + \text{encircle}①$$

$$Y = \bar{A}B\bar{C} + B\bar{C}\bar{D} + A\bar{B}\bar{D} + ABCD$$



* Don't Care Conditions :