Sahayog Sevabhavi Sanstha's

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SUPPLEMENT

Name of Student .....

Subject .....Class .....Roll No.....

Invigilator Sign .....

* Logarithms *

⊛ Product Rule :-

$$\log_a(m \cdot n) = \log_a(m) + \log_a(n)$$

or

$$\log_a(m) + \log_a(n) = \log_a(m \cdot n).$$

* Quotient Rule :-

$$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$$

or

$$\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right).$$

* Power Rule :-

$$\log_a(m)^n = n \log_a m$$

or

$$n \log_a m = \log_a(m)^n.$$

* Change of Base :-

$$\log_a m = \frac{\log_n m}{\log_n a}$$

or

$$\frac{\log_n m}{\log_n a} = \log_a m \quad .$$

Q.1) Convert the exponential form to logarithmic form $6^0 = 1$.

(2) Convert the logarithmic form to exponential form.

$$\log_4\left(\frac{1}{2}\right) = -\frac{1}{2}.$$

(3) Find the value.

$$\log\left(\frac{2}{3}\right) + \log\left(\frac{4}{5}\right) - \log\left(\frac{8}{15}\right)$$

(4) Find the value of x if

$$\log_3(x+5) = 4.$$

(5) Find the value of $\log_3(81) = ?$

(6) Prove that; $\frac{1}{\log_3 6} + \frac{1}{\log_8 6} + \frac{1}{\log_9 6} = 3.$

(7) Find the value of

(a) $\log_5 625$

(b) $\log_9(\log_2 2)$

(8) (c) Find the value of x if

(a) $\log_5(x^2 - 5x + 11) = 1.$

(b) $\log_3(x+6) = 2.$

(9) Solve: $\log_3(x+5) - \log_3(2x-1) = 2.$

* MATRICES *

* Addition, substraction & multiplication of matrices :-

① if $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

Find $2A - 3B$.

② if $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ Find $2A + 3B - 4I$.

where, I is the identity or unit matrix.

③ if $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ then

Find the matrix 'X' such that $2X + 3A - 4B = I$. where I is identity matrix of order 2.

④ if $A = \begin{bmatrix} 3 & 4 & -2 \\ 2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$

Find AB & BA.

⑤ If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ Prove That $A^2 = I$

⑥ If $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x & 5 & -3 \\ 2 & y & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 7 \\ 7 & 7 & 1 \end{bmatrix}$

Then Find the value of x & y.

⑦ Null matrix, Scalar matrix, Symmetric matrix, Skew-Symmetric matrix, Singular matrix, non-singular matrix & orthogonal matrix :-

① If $A = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$;

Show That AB is Null matrix.

② If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ Show That

$A^2 - 8A$ is a scalar matrix.

③ Find x & y if

$$\left\{4\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 & 1 \\ 2 & -3 & 4 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

④ If $A = \begin{bmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{bmatrix}$, Find $A^2 - 3A + I$.

⑤ Show that; matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

* Transpose of a matrix:-

⑥ If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Then verify that $(AB)' = B'A'$ (' is transpose of a matrix).

⑦ If $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$

verify that, $(AB)^T = B^T A^T$.

* Determinant of a matrix, singular matrix & non-singular matrix:-

① If $A = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$

Then verify 'AB' is singular or non-singular matrix.

② If $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & -3 \\ 0 & -1 & 1 \end{bmatrix}$ Find $|A|$ &

verify that A' is singular or non-singular

(3) Express the matrix 'A' as the sum of symmetric & skew-symmetric matrices. where $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$

(4) Find 'x' if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0.$

(5) If $P = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ -2 & 1 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$,

Find matrix 'R' such that $P + Q + R = 0$.

* Adjoint of a matrix :-

(1) If $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$ Find adj A.

(2) Find Adjoint of a matrix A if.

$$A = \begin{bmatrix} -1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

(3) Find Adjoint of a matrix; if

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

★ Inverse of a matrix by Adjoint method :-

(1) Find the inverse of a matrix by Adjoint method if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

(2) Find the inverse of a matrix by using Adjoint method if

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

★ Solution of a simultaneous equation by matrix inversion method :-

(1) Solve the following equations by matrix inversion method.

(a) $x + 3y + 3z = 12,$
$x + 4y + 4z = 15.$
$x + 3y + 4z = 13.$

(b) $x + y + z = 3$
$3x - 2y + 3z = 4$
$5x + 5y + z = 11$

(c)
$$x + y + z = 6$$
$$3x - y + 3z = 10$$
$$5x + 5y - 4z = 3$$

(d)
$$x + y + z = 3,$$
$$x + 2y + 3z = 4,$$
$$x + 4y + 9z = 6$$

(e)
$$2x + y = 3,$$
$$2y + 3z = 4,$$
$$2x + 2z = 8.$$

* PARTIAL FRACTION :-

Case I :- When the denominator contains non-Repeated linear Factors :-

* **Method :-**

If $(x-\alpha)$, $(x-\beta)$, etc. are the linear factors then.

(a) Write $\frac{ax+b}{(x-\alpha)(x-\beta)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)}$ - (1)

If factors are two (1).

* If factors are three then

$$\frac{ax+b}{(x-\alpha)(x-\beta)(x-\gamma)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)} + \frac{C}{(x-\gamma)}$$

Step II :- Take cross multiplication we get.

$$\frac{ax+b}{(x-\alpha)(x-\beta)} = \frac{A(x-\beta) + B(x-\alpha)}{(x-\alpha)(x-\beta)}$$

$\therefore ax+b = A(x-\beta) + B(x-\alpha)$.

Step III :- Find value of A & B & put Again in eq^{n} (1).

Questions:

Resolve into partial fractions

(a) $\frac{1}{x^2-1}$

(b) $\frac{x}{x^2+x-2}$

(c) $\frac{2x+3}{x^2-2x-3}$

(d) $\frac{x+3}{(x-1)(x+1)(x+5)}$

(e) $\frac{3x-1}{(x-4)(2x+1)(x-1)}$

(f) $\frac{2x-1}{(x+2)(x^2-1)}$

(g) $\frac{x^2+1}{x(x^2-1)}$.

* Case-II :- When the denominator contains repeated linear factors :-

Method :-

(a) $\frac{px+q}{(x+a)^2} = \frac{A}{x+a} + \frac{B}{(x+a)^2}$ or

(b) $\frac{ax+b}{(x-\alpha)^2(x-\beta)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\alpha)^2} + \frac{C}{x-\beta}$

Questions:

(1) Resolve into partial fractions

(a) $\frac{2x+1}{x^2(x+1)}$

(b) $\frac{x^2}{(x+1)(x+2)^2}$

(c) $\frac{3x+2}{(x+1)(x^2-1)}$

(d) $\frac{2x-3}{(x^2-1)(x+1)}$

Case-III :

When the denominator contains Non-Repeated irreducible linear Factors

* Resolve into partial fractions

(a) $\frac{2x+1}{(x-1)(x^2+1)}$

(b) $\frac{3x-2}{(x+2)(x^2+4)}$

(c) $\frac{x^2+1}{(x+1)(x^2+4)}$

(d) $\frac{x^2+23x}{(x+3)(x^2+1)}$

(e) $\frac{x^2-x+3}{(x-2)(x^2+1)}$

(f) $\frac{x^2+1}{x3+1}$

Case-IV :- Improper Fraction :-

(a) Resolve into partial fraction

(a) $\frac{x^4}{x^3+1}$

* STRAIGHT LINE *

* **{Slope of a line}:-**

* दोन point i.e. (x_1, y_1) आणि (x_2, y_2) मधून जाणाऱ्या line चा slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ असतो

* जर line $ax + by + c = 0$ या form मध्ये दिली असेल तर या line चा slope चा formula.

$$m = \frac{-a}{b}$$ असतो.

Question:

(1) Find the slope of a line passing through the points $(-1, -2)$ & $(-3, 8)$

(2) Find the slope of a line $2x + 3y + 6 = 0$.

(3) if the slope of a line passing through the point $(4, k)$ & $(-2, -5)$ is '2' then find 'k'

* Different form of a equation of line :-

* Formula :-

जर line चा slope दिला असेल आणि point (x_1, y_1) दिला असेल तर त्या line च्या equation चा formula

$$y - y_1 = m(x - x_1)$$ असतो.

Question

(1) Find the equation of line passing through (3, -4) & having slope $\frac{3}{2}$

(2) Find the equation of line passing through points (2, 3) & having slope is '5' units.

(2) Formula :-

जर दोन point दिले असतील i.e. (x_1, y_1) आणि (x_2, y_2) तर त्या line च्या equations चा Formula

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

Questions

(1) Find the equation of line passing through the points (-4, 6) & (8, -3)

(ii) Find the equation of line passing through the points (3, 5) & (4, 6).

* Angle between two lines :-

- Formula :-

The Acute Angle θ between two intersecting lines is

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right| \Rightarrow \theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

where,

m_1 - is slope of a line l_1

m_2 - is slope of a line l_2.

Questions:

(1) Find the acute angle between the lines $3x - y = 4$ & $2x + y = 3$.

(2) Find the acute Angle between the lines $3x - 2y + 4 = 0$ & $2x - 3y - 7 = 0$.

(3) Find the ~~equa~~ acute Angle between the lines whose slopes are $\sqrt{3}$ & $\frac{1}{\sqrt{3}}$.

* **Condition of parallel & perpendicular lines :-**

(1) Find the equation of line passing through the points (4,5) & perpendicular to the line $7x - 5y = 420$.

(2) Find the equation of line passing through the point (2,3) & perpendicular to the line $3x - 5y = 6$.

(3) Find the equation of line passing through (2,-3) & parallel to the line $4x - y + 7 = 0$

(4) Find the equation of perpendicular Bisector of line joining the points A(-2,3) & B(8,-1).

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(VIMP) Intersection of two lines :-

(1) Find the equation of a straight line passing through the points of intersection of lines $4x+3y=8$ & $x+y=1$ & parallel to the line $5x-7y=3$.

(2) Find the equation of line passing through the point of intersection of lines $x+y=0$ & $2x-y=9$ and through the points $(2,5)$.

(3) Find the equation of the line passing through the point of intersection (VIMP) of the line $2x+3y=13$, $5x-y=7$ & perpendicular to $3x-y+7=0$.

* Length of perpendicular

- Formula :- $\left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|$

(1) Find the length of perpendicular from the point $P(2,5)$ on the line $2x+3y-6=0$.

(2) Find the length of perpendicular from the point $P(5,4)$ on $2x+y=34$.

③ Find the lengths of perpendicular from $(-3, -4)$ on $4(x+2) = 3(y-4)$.

* **Distance between two parallel lines:**

Formula :- $$\left| \frac{c_1 - c_2}{\sqrt{a^2+b^2}} \right|$$

① Find the distance between the parallel lines $3x - y + 7 = 0$ & $3x - y + 16 = 0$.

② Find the distance between the parallel lines $3x + 2y - 5 = 0$ & $3x + 2y - 6 = 0$.

* **Intercepts of a line on Axes :-**

equation :- $$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

① Find the intercepts of the line $2x + 3y = 6$ on both Axes.

② Find the intercepts of the line $y = 3x - 4$ on co-ordinate axes.

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* Statistics *

* 2-marks *

* **Range:**

First arrange the data in ascending order.

Formula:-

$$\text{Range} = L - S$$

Where,

L - Largest item

S - Smallest item.

* **Coefficient of Range :-** $= \frac{L-S}{L+S}$.

Questions:

(1) Find the range & coefficient of range for the data

3, 7, 11, 2, 16, 17, 22, 20, 19.

(2) Find the range & coefficient of range for the data;

45, 42, 39, 40, 48, 41, 45, 44.

(3) Find the range & coefficient of range of following:

x_i	10	20	30	40	50.
f_i	7	5	3	2	1

④ Find the Range & coefficient of Range of following data

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Stud.	10	15	16	20	21	22	9	8

⑤ Calculate the Range & coefficient of Range from the following data.

Class Interval	10-19	20-29	30-39	40-49	50-59	60-69
Freq.	6	10	16	14	8	4.

(4 marks)

(mean deviation):-

① Calculate the mean deviation about the mean of the following data: 3, 6, 5, 7, 10, 12, 15, 18.

② Calculate the mean deviation about the mean of the following data: 17, 15, 18, 23, 25, 22, 11, 5.

③ Calculate the mean deviation about the mean of following data: 12, 6, 7, 3, 15, 10, 18, 5.

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* Mean deviation for ungroup data:-

(1) Find the mean deviation from the mean of the following data.

mark	0-10	10-20	20-30	30-40	40-50
no.of stu	5	8	15	16	6.

(2) Calculate the mean deviation about the mean of the following data:

mark	3	4	5	6	7	8		
no. of studn	1	3	7	5	2	2.		

(3) Find the mean deviation from median

weight	10-15	15-20	20-25	25-30	30-35	35-40	40-45
no of item	7	12	16	25	19	15	6.

* Standard deviation:-

* 2 mark *

(1) If mean is 34.5 & standard deviation is 5. Find the coefficient of variance.

(2) If coefficient of variation is 75% & standard deviation is 24, find its mean.

③ If coefficient of variation of certain data is 5 & mean is 60. Find the standard deviation.

* 4 marks *

① Compute the standard deviation for 15, 22, 27, 11, 9, 21, 14, 9.

② Compute the standard deviation for 1, 2, 3, 4, 5, 6, 7.

* Standard deviation for ungroup data:

① Calculate the standard deviation & coefficient of variance of the following table.

mark below	5	10	15	20	25
no. of studn	6	16	28	38	46.

* Standard deviation for group data:-

(VIMP)

② Calculate the mean, standard deviation, variance, coefficient of standard deviation & coefficient of variation for the following data.

Expenditure	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15.

③ Calculate the mean & variance of following

class intervals	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	8	3	1.

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* Comparison of two sets of observations:-

* 2 marks :-

(1) The two sets of observations are given below

Set - I	Set - II
$\bar{X} = 82.5$	$\bar{X} = 48.75$
C.V) $6 = 7.3$	(S.D) $6 = 8.35$.

Which of two is more consistent.

(2) In two factories A & B, engaged in the same industry, in the area, the Average weekly wages (Rs) & the S.D. are as follows.

Factory	Average	S.D.
A	34.5	5.0
B	28.5	4.5.

Which factory of A or B is more consistent. ?

(3) The following data pertain to two workers doing the same job in a factory:

	worker A	worker B.
mean time	40	42
standard deviation.	8	6.

✱ Functions & Limits :-

① If $f(x) = x^4 - 2x + 7$. Find $f(0) + f(2)$

② If $f(x) = x^3 - 3x^2 + 5$, Find $f(0) + f(3)$

③ If $f(x) = 3x^2 - 5x + 7$; Show that $f(-1) = 3 \cdot f(1)$.

④ If $f(x) = x^2 - 2x + 5$ & $t = y - 2$; Find $f(t)$.

⑤ If $f(x) = x^3 - \frac{1}{x^3}$ Then show that

$$f(x) + f\left(\frac{1}{x}\right) = 0.$$

✱ odd or even function :-

① Define odd & even function with suitable example.

② State wheather the function $f(x) = \frac{e^x + e^{-x}}{2}$ is odd or even.

③ State wheather the function $f(x) = \frac{a^x + a^{-x}}{2}$ is odd or even.

④ If $f(x) = 3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x$ show that $f(x)$ is an even function.

⑤ Test wheather the function is even or odd if
$f(x) = x^3 + 4x + \sin x$.

* Applications of derivative *

Tangent & normal :-

(1) Find the equation of tangent & normal to the curve $y = x(2-x)$ at the point $(2, 0)$.

(2) Find the equation of tangent & normal to the curve $4x^2 + 9y^2 = 40$ at $(1, 2)$.

(3) Find the equation of tangent & normal to the curve $2x^2 - xy + 3y^2 = 18$ at $(3, 1)$.

(4) Find the point on the curve $y = 7x - 3x^2$ where tangent makes Angle of $45°$.

(5) Find the equation of tangent & normal to the curve

$$x = \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad \text{when } t = 2.$$

* **Maxima & Minima :-**

(1) Find the maximum & minimum values of $x^3 - 9x^2 + 24x$.

(2) Find the maximum & minimum values of $2x^3 - 3x^2 - 12x + 12$

(3) Find the maximum & minimum values of $y = 2x^3 - 3x^2 - 36x + 10$.

(4) A metal wire 36 cm long is bent to form a rectangle. Find its dimension when its area is maximum. (VIMP)

(5) Divide 100 into two parts such that there product is maximum. (VIMP)

(6) Divide 80 into two parts such that their product is maximum.

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Radius of curvature :-

Formula :-

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

(1) Find Radius of curvature of $y = x^3$ at point $(1, 1)$

(2) Find Radius of curvature of $y = x^3$ at point $(2, 1)$

(3) Find Radius of curvature of $y = e^x$ at $(0, 1)$

(4) Find Radius of curvature of the curve $y^2 = 4ax$ at $(a, 2a)$.

(5) VIMP A beam is bent in the form of curve $y = 2\sin x - \sin 2x$. Find Radius of curvature of the beam at $x = \frac{\pi}{2}$.

(6) VIMP A Telegraph wires hangs in the form of a curve $y = a \log\left[\sec \frac{x}{a}\right]$, a is const

Show that the radius of curvature at any point is $a \sec\left(\frac{x}{a}\right)$. And

Show that the curvature at any point is $\frac{1}{a}\cos\left(\frac{x}{a}\right)$.